# 8 - Convolutional Networks

**Ludwig Krippahl**

FACULDADE DE CIÊNCIAS E TECNOLOGIA UNIVERSIDADE NOVA DE LISBOA

## Summary

- What is convolution
- Convolution layers and networks
- Pooling
- Classification with convolutional networks
- Introduction to the Keras Sequential API
- CNN tutorial: Fashion MNIST classification with CNN and Keras

# Convolution

# Convolution

**Definition:**

- Integral of the product of two functions, one of which was shifted and inverted

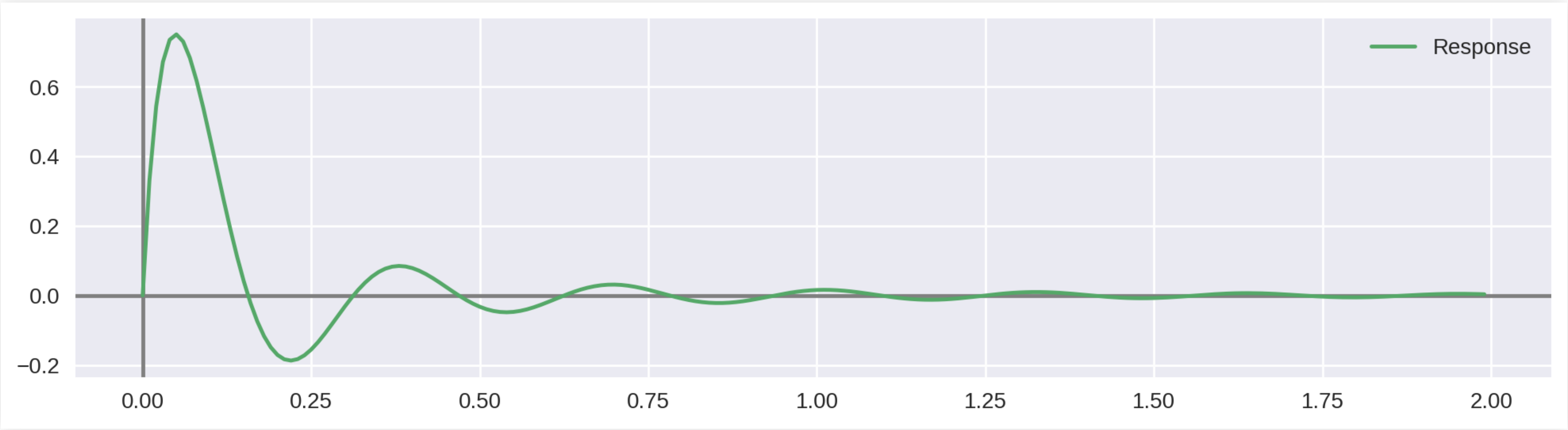
$$(f * g)(t) = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau = \int_{-\infty}^{\infty} f(t-\tau)g(\tau)d\tau$$

- Used in many applications, such as probabilities or linear time-invariant systems

# Convolution

$$(f * g)(t) = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau = \int_{-\infty}^{\infty} f(t-\tau)g(\tau)d\tau$$

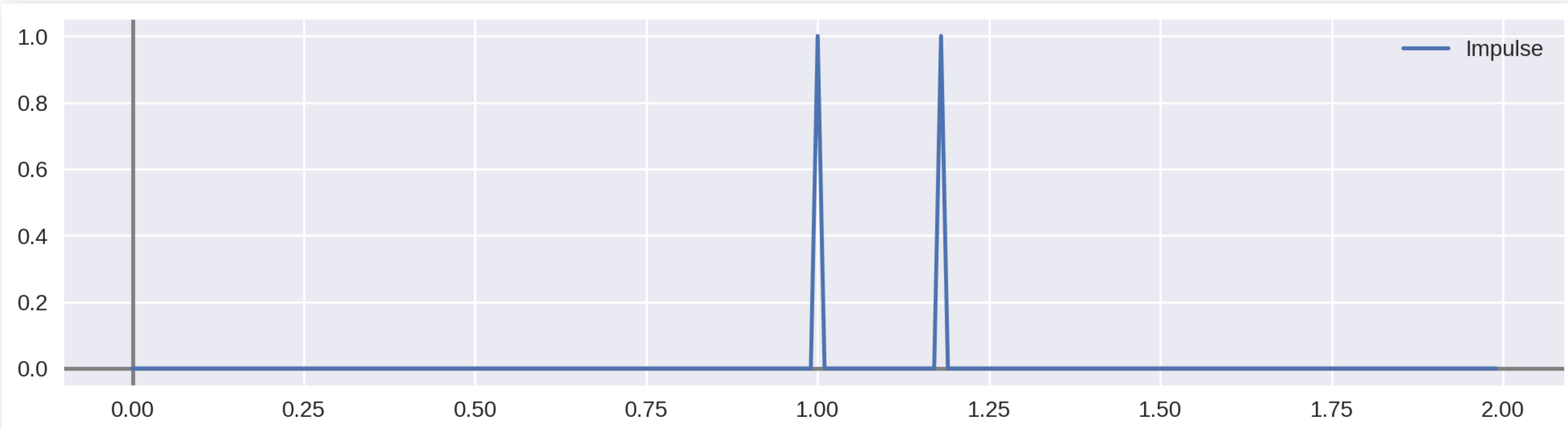
- Intuition: Suppose a LTIS with this response:



# Convolution

$$(f * g)(t) = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau = \int_{-\infty}^{\infty} f(t-\tau)g(\tau)d\tau$$

- Subject to this impulse

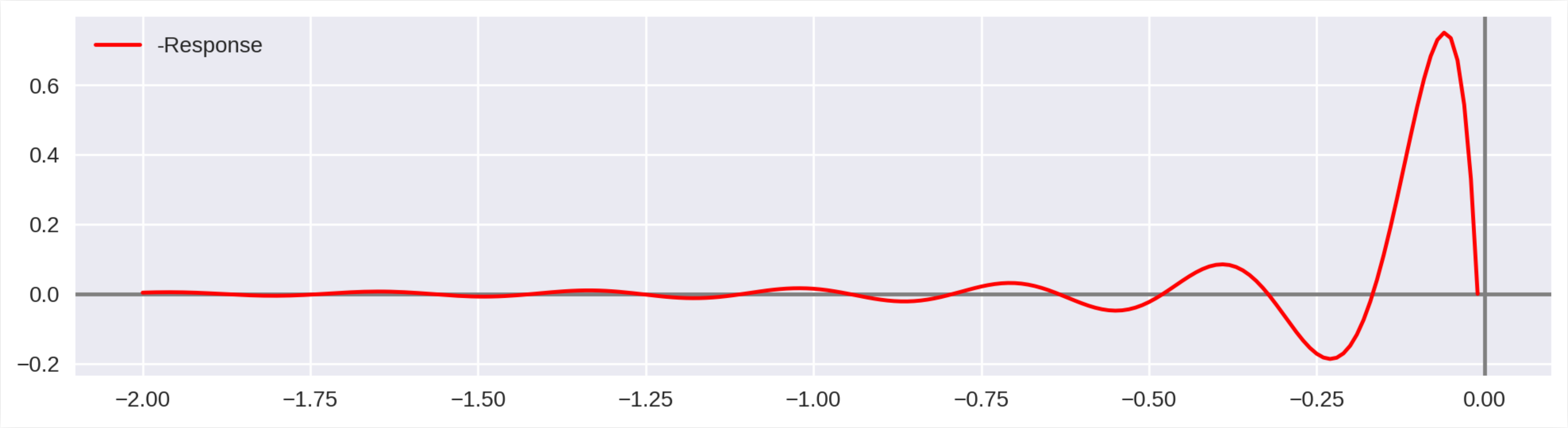



- How do we compute the output?

# Convolution

$$(f * g)(t) = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau = \int_{-\infty}^{\infty} f(t-\tau)g(\tau)d\tau$$

- First we take the symmetric of the response:

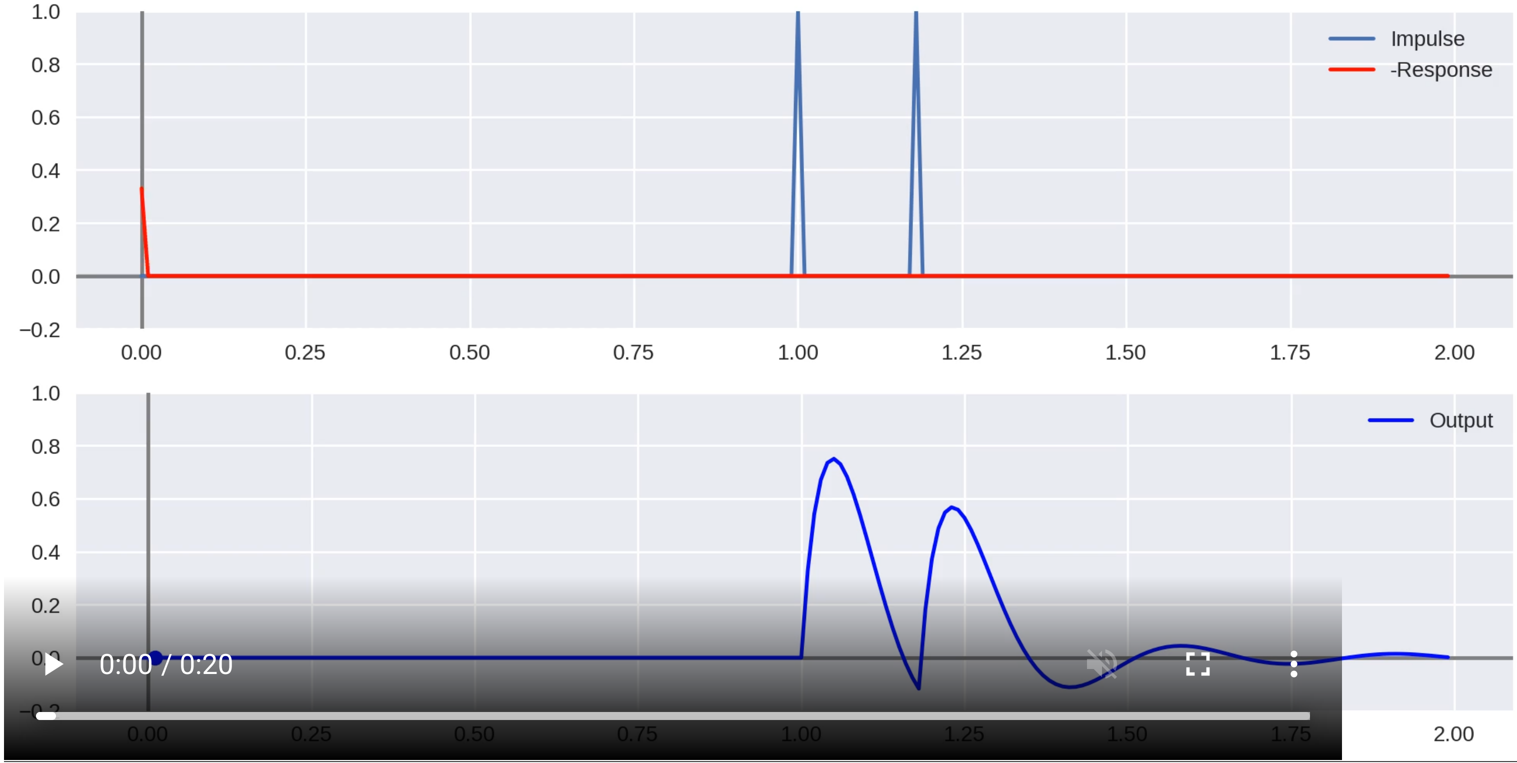



- How do we comput response?

# Convolution

$$(f * g)(t) = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau = \int_{-\infty}^{\infty} f(t-\tau)g(\tau)d\tau$$

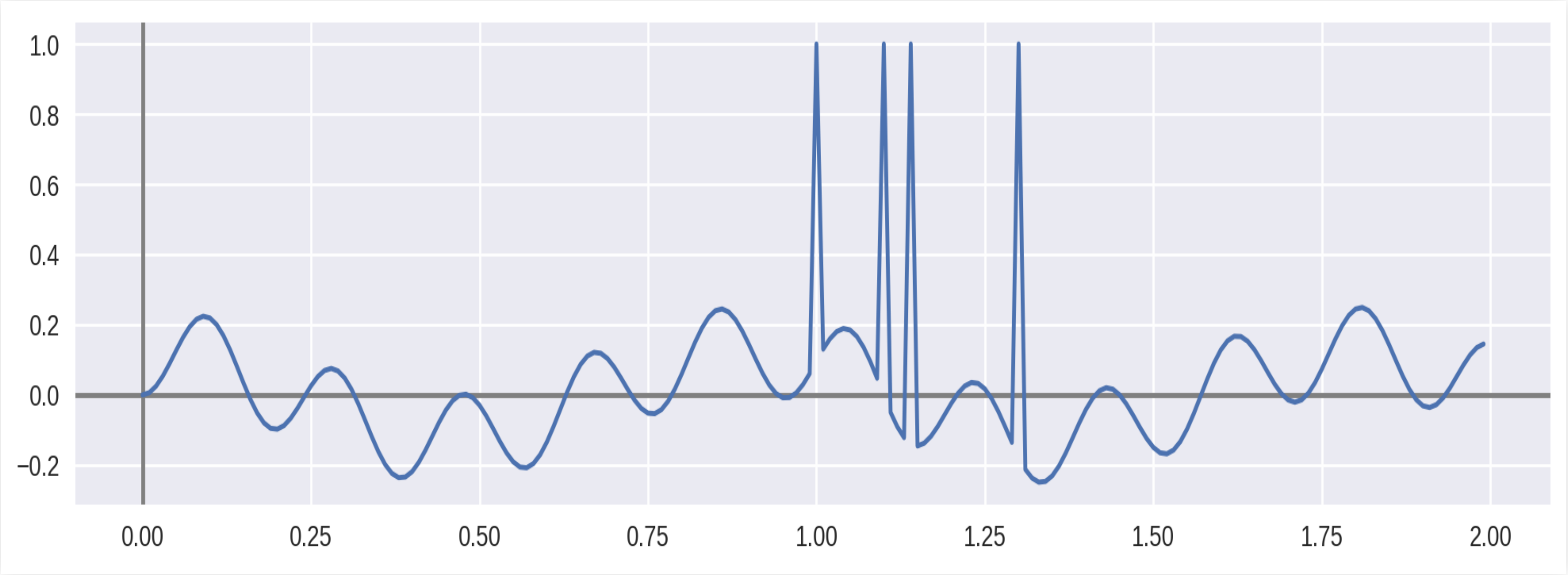
- Now we integrate product at different time shifts



# Convolution

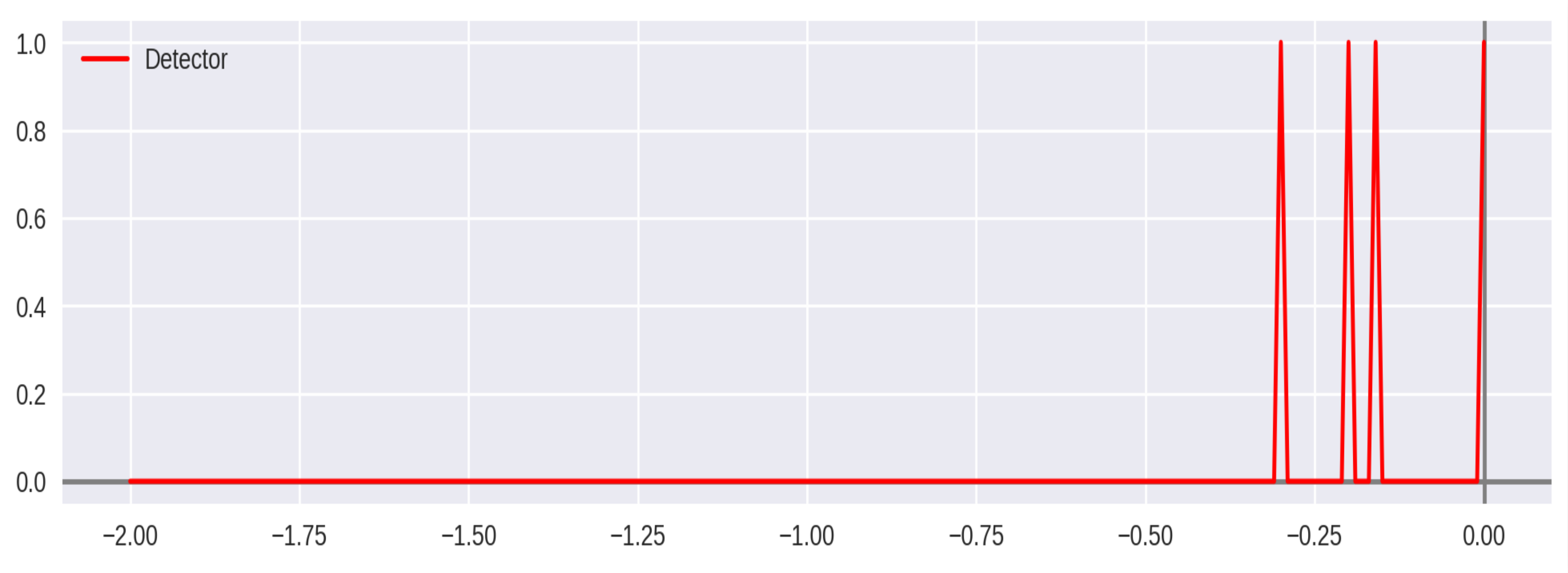
## For finding patterns

- Suppose we have this signal and want to detect the spike pattern



## For finding patterns

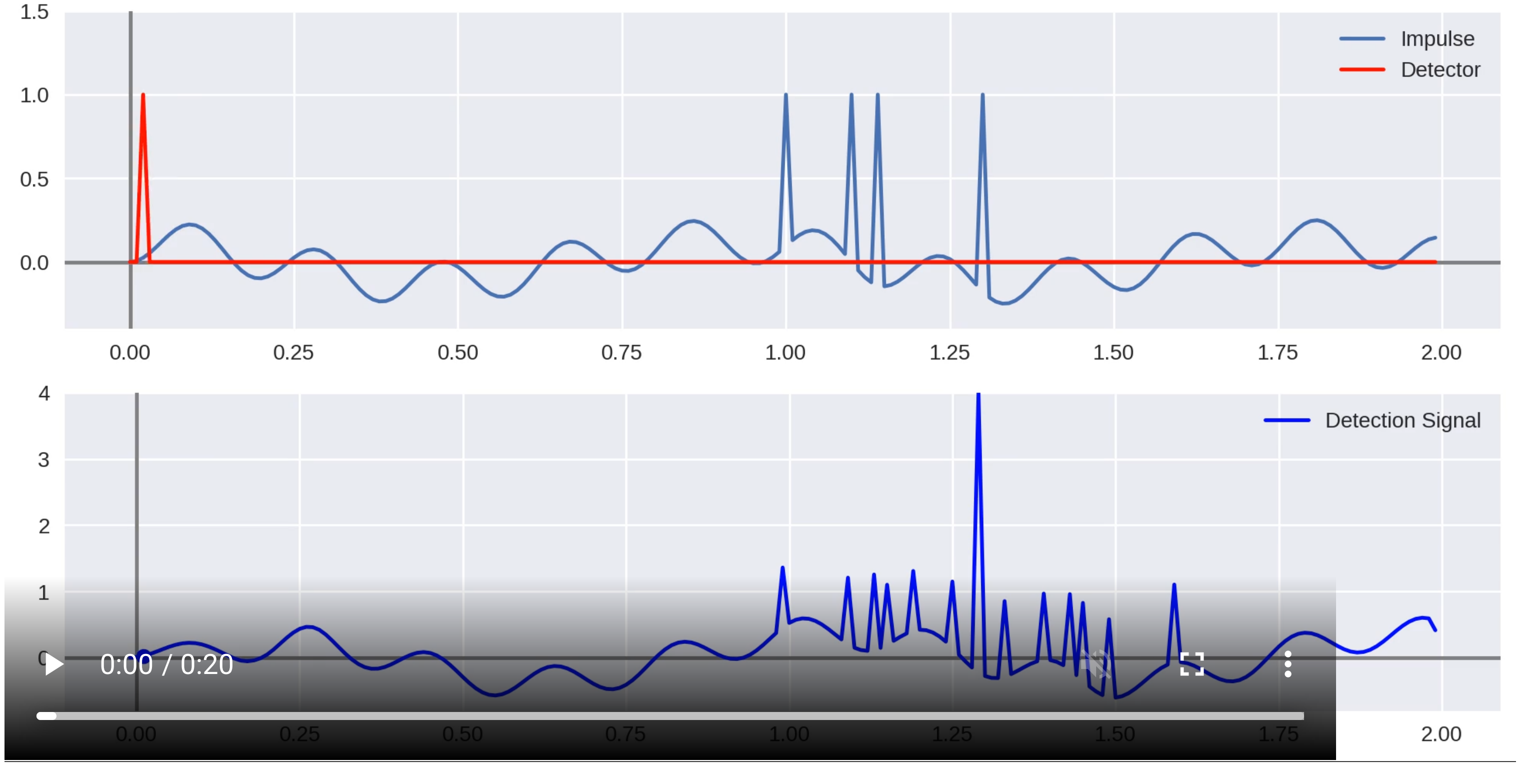
- We create this "detector" function (inverted)



# Convolution

$$(f * g)(t) = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau = \int_{-\infty}^{\infty} f(t-\tau)g(\tau)d\tau$$

- We detect the pattern integrating at different time shifts



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# Convolution

## Definition:

- Integral of the product of two functions, one of which was shifted and inverted

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## Discrete convolution:

$$(f * g)(n) = \sum_{m=-\infty}^{\infty} f(m)g(n-m) = \sum_{m=-\infty}^{\infty} f(n-m)g(m)$$

# Convolution

## Motivation:

- Use a *kernel* to modify the *input* and create a new function using weighted values "around" the input

- E.g. a weighted average of most recent values in a time series, local features of an image, etc,

$$s(t) = (x * w)(t) = \sum_{m=-\infty}^{\infty} x(t)w(t-m)$$

- In practice, for CNN:

- The input is a finite, discrete set of values (assumed 0 everywhere else)
- The kernel is a finite set of parameters that will be learned
- The output will be tensor, tipically of the same size and shape as the input

## Two dimensional convolution

- We often use convolution in more than one dimension (e.g. for images)

$$S(i,j) = (I * K)(i,j) = \sum_m \sum_n I(m,n)K(i-m, j-n)$$

- Since convolution is commutative, we often use:

$$S(i,j) = (I * K)(i,j) = \sum_m \sum_n I(i-m, j-n)K(m,n)$$

- I.e. we index the kernel and the image around (i,j)

## Two dimensional convolution

- Since the "direction" of the index is only for commutativity, we actually use the cross-correlation function (but it doesn't matter)

$$S(i,j) = (I \star K)(i,j) = \sum_m \sum_n I(i+m, j+n) K(m,n)$$

- The difference is only whether we flip the kernel or not
- So, generally, in machine learning convolution and cross-correlation are both called convolution

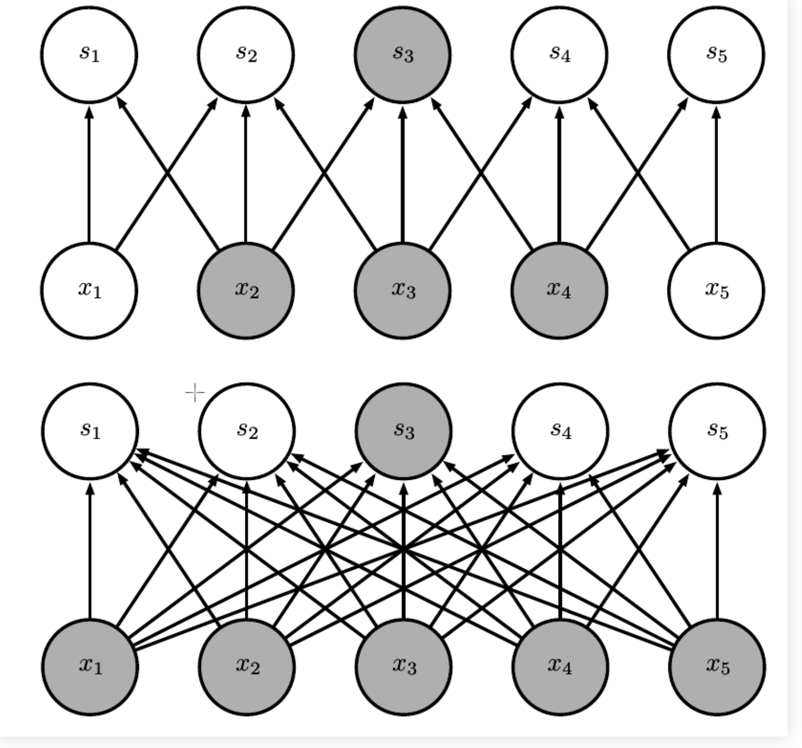
# CNN

## Motivation for convolutional networks

- Sparse interactions (sparse connectivity or sparse weights)
  - Since the kernel is smaller than the input, we need fewer parameters
- Shared parameters
  - Connections are sparse but the same parameters are used over all inputs
- Equivariance
  - Convolutions are equivariant to some transformations
  - I.e. applying the transformation (e.g. translation) to the input is the same as applying it to the convolution

## Motivation: sparse interactions

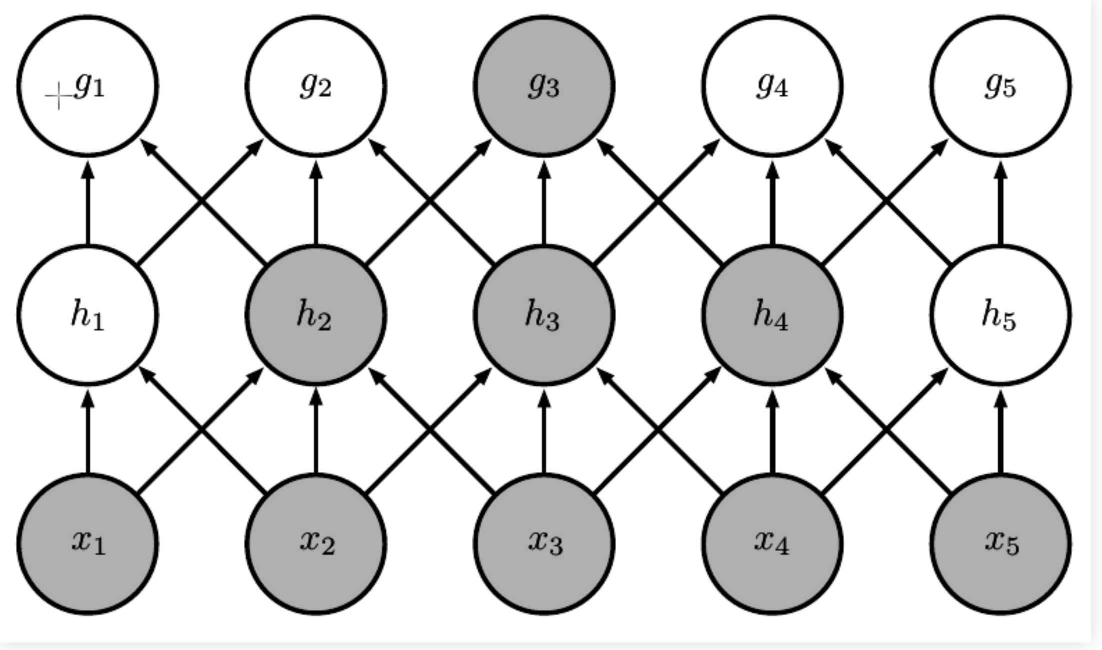
- Convolutional networks have fewer connections than MLP




Goodfellow, Bengio, Courville, Deep Learning 2016

## Motivation: sparse interactions

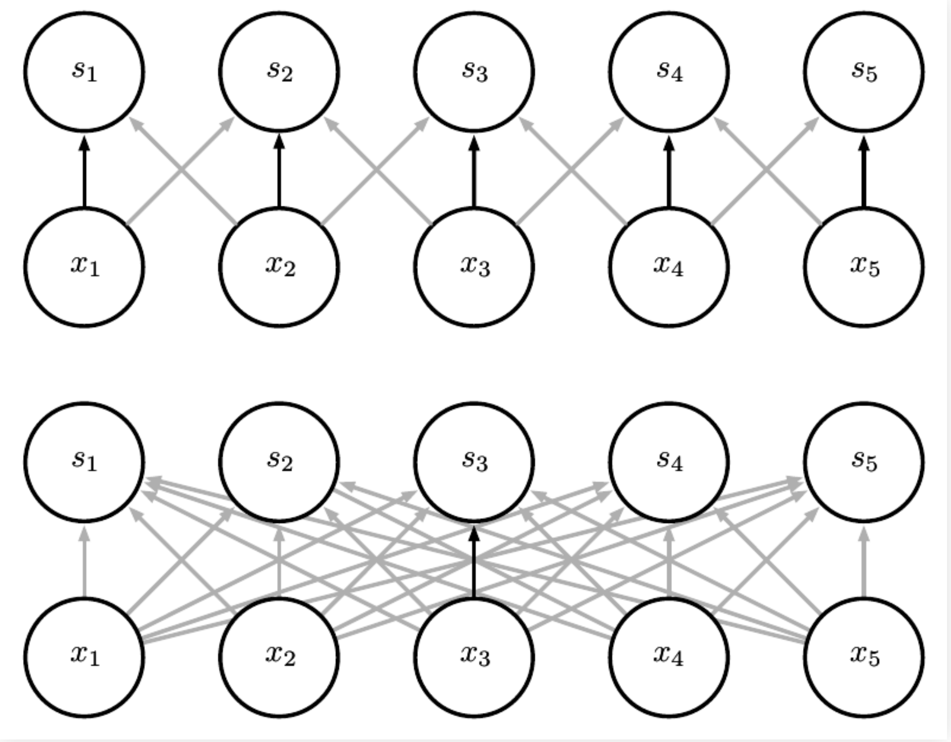
- Convolutional networks have fewer connections than MLP
- But deeper neurons can still have a large receptive field in the input




Goodfellow, Bengio, Courville, Deep Learning 2016

## Motivation: parameter sharing

- The same parameter is used for many inputs




Goodfellow, Bengio, Courville, Deep Learning 2016

## Motivation: parameter sharing

- The same parameter is used for many inputs
  - E.g. edge detection by subtracting pixel on the left

Goodfellow, Bengio, Courville, Deep Learning 2016

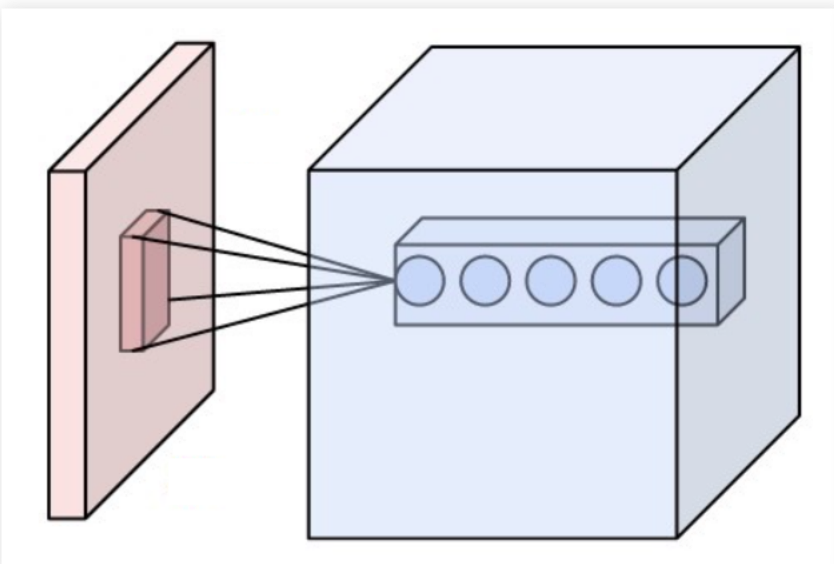
## Motivation: equivariance

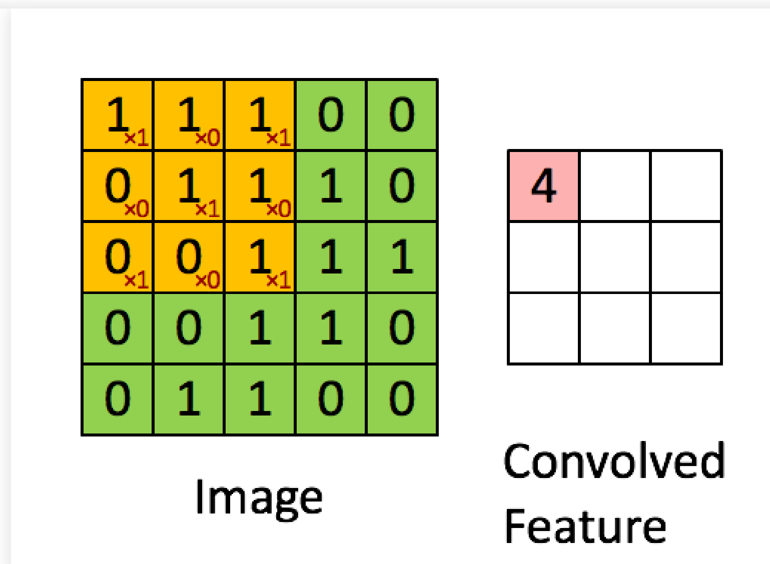
- Moving the input image is equivalent to moving the output of the convolution filter (the *feature map*)

Goodfellow, Bengio, Courville, Deep Learning 2016

## Receptive field and multiple filters

- Visual cortex neurons respond to a small receptive field in retina
  - Neurons in convolution layers have this property
- Multiple convolution filters are generally applied






Images: Aphex34, CC-SA; UFLDL tutorial, Stanford (Ng et. al.)

## Receptive field and multiple filters

- Multiple convolution filters are generally applied
  - We generally represent convolution layers as 3D volumes of neurons (in image processing), stacking the different convolution filters

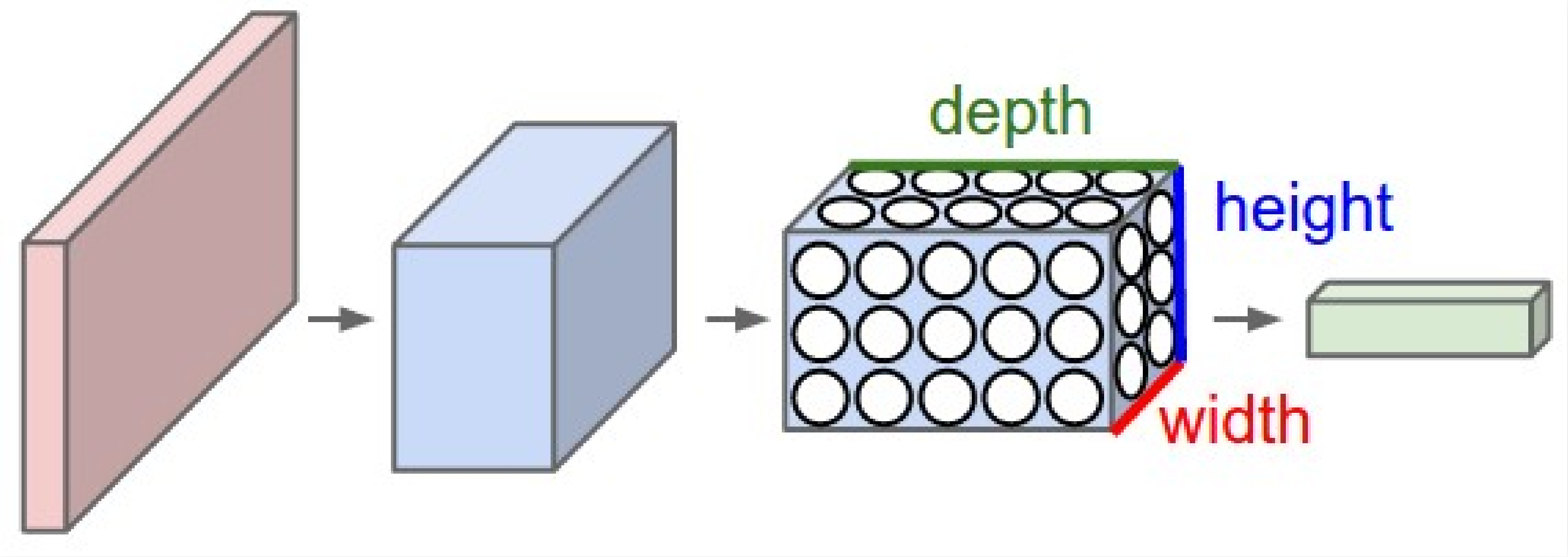
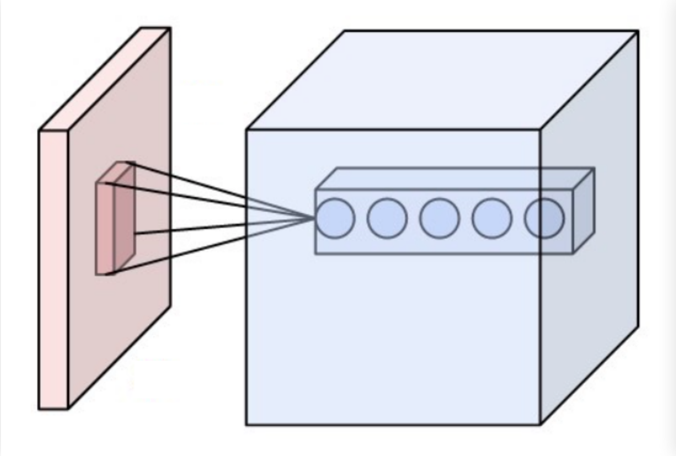
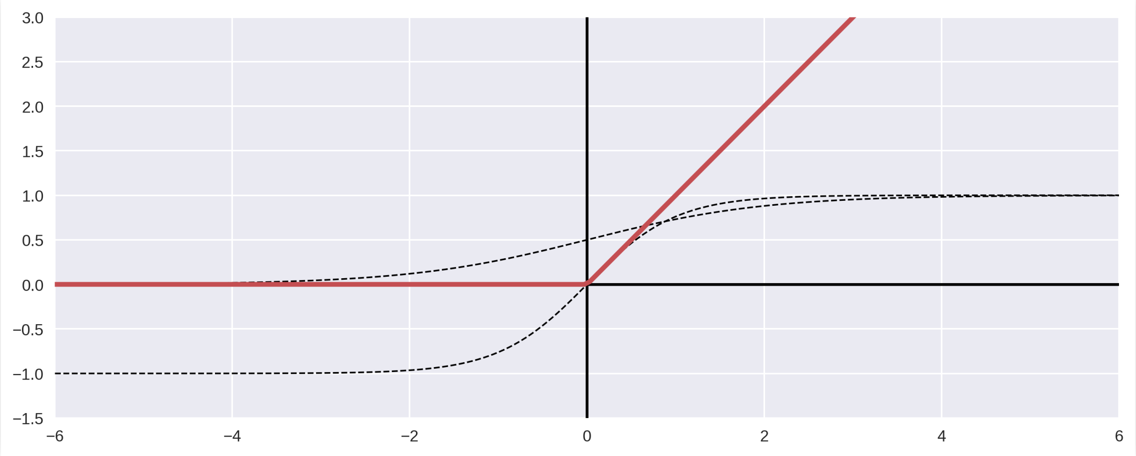



Image: CS231n Stanford (Fei-Fei Li et. al.)

## Nonlinear tranformation

- A convolution is a linear transformation, so in CNN the convolution filter generally feeds into a nonlinear response (usually ReLU)





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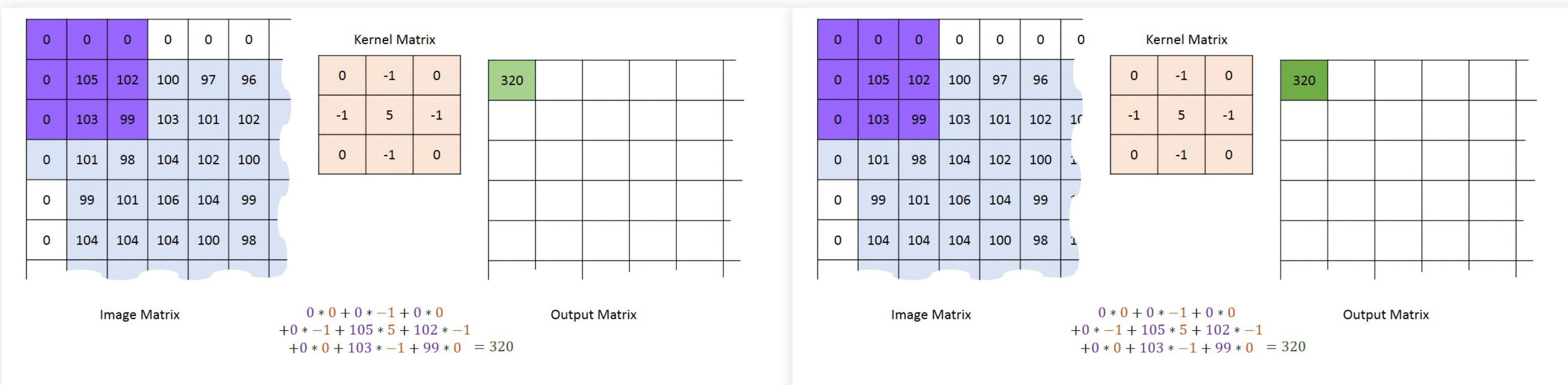
# Hyperparameters of a convolution layer

- Depth: the number of filters being learned
- Stride: How the receptive field "jumps"
- Padding: to prevent output from shrinking




**Convolution with horizontal and vertical strides = 1**

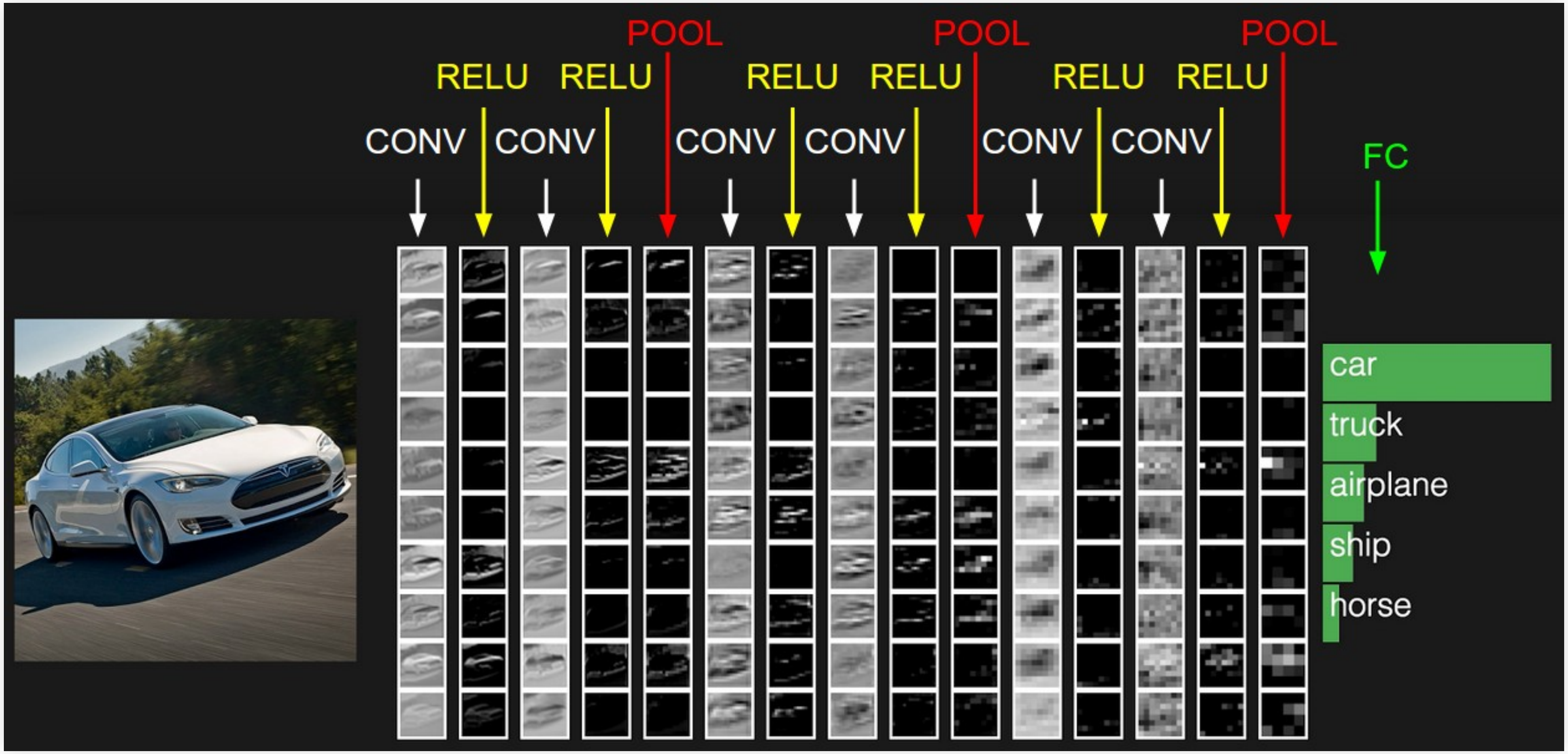
**Convolution with horizontal and vertical strides = 2**

Image: Machine learning guru, http://machinelearninguru.com

## Example: convolution network

- Stanford (CS231, Fei-Fei Li et. al.)

• http://cs231n.github.io/convolutional-networks/



# Pooling

# Pooling

## Typical architecture of a convolutional layer

- Convolutions: several convolutions in parallel, generating a set of linear activations
- Nonlinear activation: applied to each linear output (tipically ReLU)
- (detector stage)
- Pooling: aggregates the outputs in a single output for each region

## Pooling: aggregating outputs

- Example: max pooling

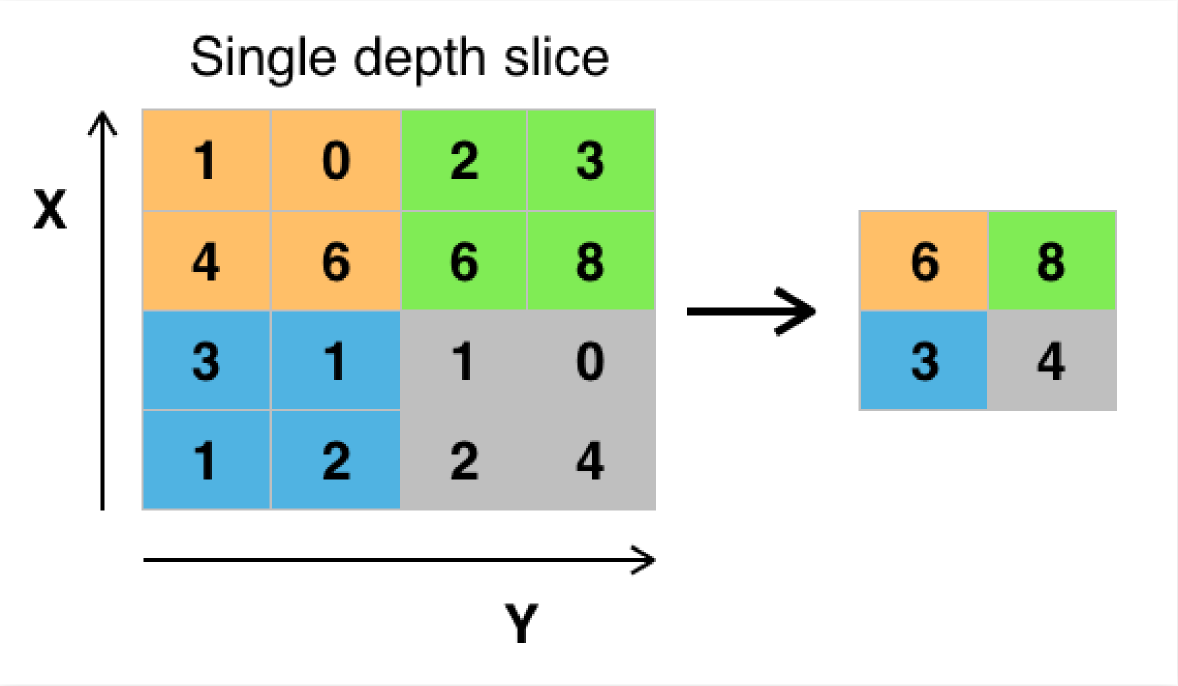



Image: Aphex34, CC-SA

# Pooling

## Pooling: aggregating outputs

- Typical pooling functions:
  - Max pooling, average pooling, $L^2$ norm pooling: $\sqrt{\sum x_i^2}$
- Pooling makes model nearly invariant to small shifts in input

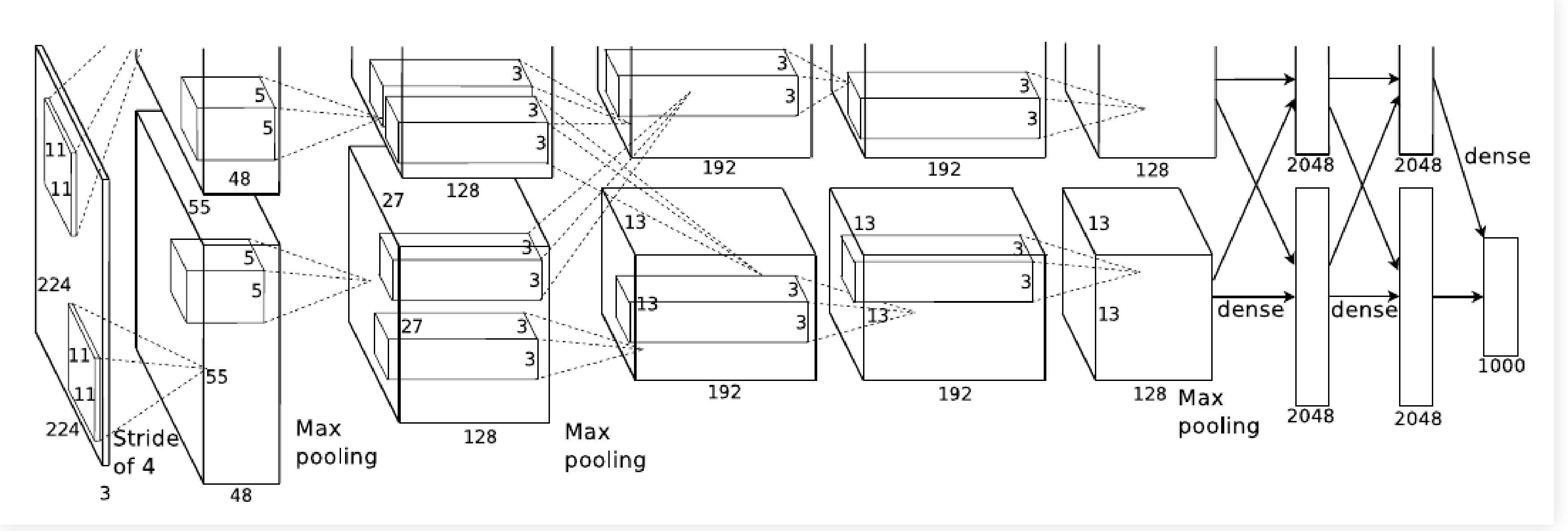
## Pooling Stride

- We may want to regulate the overlap of pooling regions
- If the stride is equal to the size of the regions, then there is no overlap
- Stride also reduces the dimension
  - (pooling of stride 1, with padding, preserves dimension)

# Classificaton with CNN

## For classification, conv layers combined with MLP

- Fully connected layers at the end to predict class for example
- Forces fixed-sized input and output not spacial




Alex Krizhevsky, Ilya Sutskever, Geoffrey E. Hinton, 2012

# Tutorial: Keras Sequential API

## Building a model with Keras

- Start by importing classes:

```
from tensorflow.keras.optimizers import SGD
from tensorflow.keras.models import Sequential
from tensorflow.keras.layers import BatchNormalization,Conv2D,MaxPooling2D,
from tensorflow.keras.layers import Activation, Flatten, Dropout, Dense
```

## Building a model with Keras

- Create a `Sequential` model and add layers

```
model = Sequential()
model.add(Conv2D(32, (3, 3), padding="same", input_shape=(28,28,1)))
model.add(Activation("relu"))
...
model.add(MaxPooling2D(pool_size=(2, 2)))
model.add(Conv2D(64, (3, 3), padding="same"))
...
model.add(Flatten())
model.add(Dense(512))
model.add(Activation("relu"))
model.add(BatchNormalization())
model.add(Dropout(0.5))
model.add(Dense(10))
model.add(Activation("softmax"))
```

# Keras Sequential

**Side note:**

- (Current consensus, but some opinions may vary)
- Use batch normalization after activation so that input of following layer is standardized
- Use dropout after all batch normalizations
  - Otherwise you may need to compensate the rescaling of dropout when shifting to prediction mode
- Dropout may have some benefits in convolution layers but in that case it is not the same as dropout in dense layers

## Compiling the model

```
opt = SGD(lr=INIT_LR, momentum=0.9, decay=INIT_LR / NUM_EPOCHS)
model = create_model()
model.compile(loss="categorical_crossentropy", optimizer=opt,
        metrics=["accuracy"])
```

- Now we can train the model and save the weights.

```
history = model.fit(trainX, trainY, validation_data=(testX, testY),
                  batch_size=BS, epochs=NUM_EPOCHS)
model.save_weights('fashion_model.h5')
```

- The saved weights can be loaded with
  - `model.load_weights(file_name)`

## Monitoring

- Check the model, after compiling

```
model.summary()
```

- Callbacks (e.g. for Tensorboard)
  - https://keras.io/callbacks/

```
tb_callback = keras.callbacks.TensorBoard(log_dir='./logs', write_graph=True)
...
history = model.fit(trainX, trainY, validation_data=(testX, testY),
                    batch_size=BS, epochs=NUM_EPOCHS,
                    callbacks = [tb_callback])
```

## Monitoring

- The `fit` method returns an history object (parameters, model, etc)

```
history = model.fit(trainX, trainY, validation_data=(testX, testY),
                    batch_size=BS, epochs=NUM_EPOCHS,
                    callbacks = [tb_callback])
```

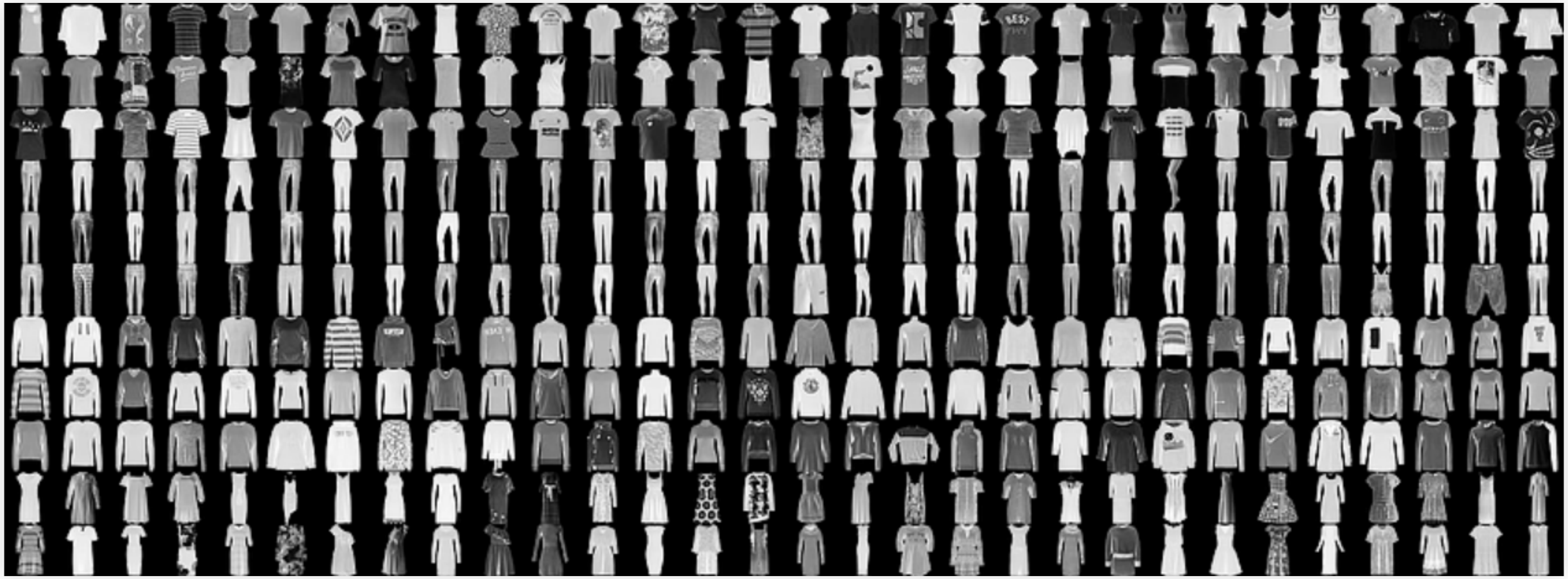
```
In: history.history
Out:
 {'acc': [0.7736167, ...],
  'loss': [0.6834171069622039, ...],
  'val_acc': [0.1974, ...],
  'val_loss': [2.1077217151641845, ...]}
```

# Tutorial: Fashion MNIST and CNN

## Fashion MNIST

https://github.com/zalandoresearch/fashion-mnist

- Grayscale images, 28x28, 10 classes of clothing



# Tutorial

## Import the dataset and set up the data

```
from tensorflow import keras
((trainX, trainY), (testX, testY)) = keras.datasets.fashion_mnist.load_data()
trainX = trainX.reshape((trainX.shape[0], 28, 28, 1))
testX = testX.reshape((testX.shape[0], 28, 28, 1))
trainX = trainX.astype("float32") / 255.0
testX = testX.astype("float32") / 255.0
# one-hot encode the training and testing labels
trainY = keras.utils.to_categorical(trainY, 10)
testY = keras.utils.to_categorical(testY, 10)
```

# Tutorial

## Model for this exercise:

- First stack:
  - Two convolution layers with $3 \times 3$ kernel, padding "same", 32 filters, ReLU activation and batch normalization
  - Max pooling of size $2 \times 2$ and same stride
  - Optional: you can try adding dropout layer with 25% dropout probability (but results seem to be worse)
- Second stack: Identical to first but with 64 filters
- Dense layer of 512 neurons with ReLU activation, batch normalization and dropout of 50%
- Softmax layer with 10 neurons.
- Use the SGD optimizer and about 25 epochs, save after fitting
- Optional: Experiment changing the model and optimizers

# Summary

# Convolutional networks

## Summary

- Convolutions
- Convolution layers
- Classification with convolutional networks
- CNN tutorial using the Keras sequential API

## Further reading:

- Goodfellow et.al, Deep learning, Chapter 9
- Tensorflow Keras API

• https://www.tensorflow.org/guide/keras