

ARCHIMEDES

Introduction

CALCULUS: A NEW HORIZON FROM ANCIENT ROOTS

*C*alculus, which is sometimes called the “mathematics of change,” is the branch of mathematics concerned with describing the precise way in which changes in one variable relate to changes in another. In almost every human activity we encounter two types of variables: those that we can control directly and those that we cannot. Fortunately, those variables that we cannot control directly often respond in some way to those that we can. For example, the acceleration of a car responds to the way in which we control the flow of gasoline to the engine, the inflation rate of an economy responds to the way in which the national government controls the money supply, and the level of an antibiotic in a person’s bloodstream responds to the dosage and timing of a doctor’s prescription. By understanding quantitatively how the variables we cannot control directly respond to those that we can, we can hope to make predictions about the behavior of our environment and gain some mastery over it. Calculus is one of the fundamental mathematical tools used for this purpose.

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CALCULUS TODAY

Calculus has an enormous, but often unnoticed, impact on our daily lives. To provide some sense of how calculus affects us, we have selected a few of its applications to fields of contemporary research. All of these applications involve other branches of science and mathematics, but they all use calculus in some essential way. The first three applications are based on a new and exciting area of mathematics called the theory of *wavelets*. Wavelets make it possible to capture and store mathematical representations of images and signals using much less data than previously possible. As a result, the current research literature is exploding with new applications of wavelets to such diverse fields as astronomy, acoustics, nuclear engineering, image processing, neurophysiology, music, medicine, speech synthesis, earthquake prediction, and pure mathematics, to name only a few.

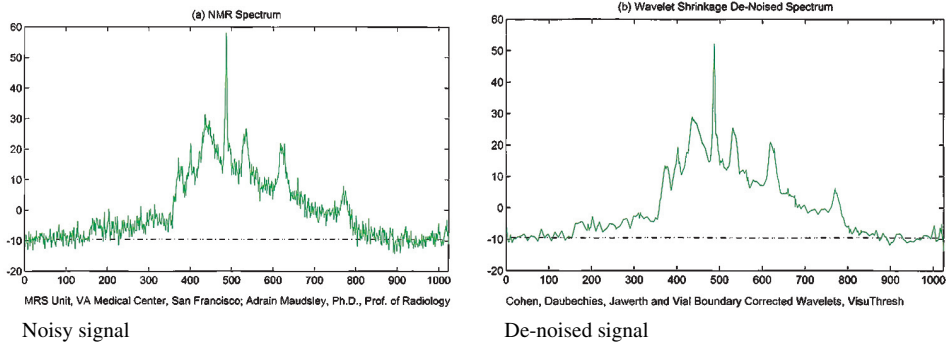
FBI Fingerprint Compression — The U.S. Federal Bureau of Investigation began collecting fingerprints and handprints in 1924 and now has more than 30 million such prints in its files, all of which are being digitized for storage on computer. It takes about 0.6 megabyte of storage space to record a fingerprint and 6 megabytes to record a pair of handprints, so that digitizing the current FBI archive would result in about 200×10^{12} bytes of data to be stored, which is the capacity of roughly 138 million floppy disks. At today's prices for computer equipment, storage media, and labor, this would cost roughly 200 million dollars. To reduce this cost, the FBI's Criminal Justice Information Service Division began working in 1993 with the National Institute of Standards, the Los Alamos National Laboratory, and several other groups to devise compression methods for reducing the storage space. These methods, which are based on wavelets, are proving to be highly successful. Figure 1 is a good example—the image on the left is an original thumbprint and the one on the right is a mathematical reconstruction from a 26:1 data compression.



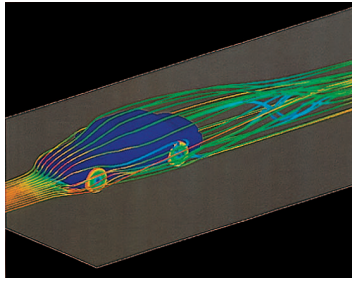
Original
Reconstruction
Figure 1

Music — Researchers with the Numerical Algorithms Research Group at Yale University have investigated the application of wavelets to sound synthesis (musical and voice). To approximate the sound of a musical instrument or voice, samples are taken and decomposed mathematically into numbers called *wavelet packet coefficients*. These coefficients can be stored on a computer and later the sound can be reconstructed (synthesized) from the computer data. This area of research makes it possible to reproduce complex sounds from a small amount of data and to transmit those data electronically in a highly compressed form. This research may eventually speed up the transmission of sound over the Internet, for example.

Removing Noise from Data — In fields ranging from planetary science to molecular spectroscopy, scientists are faced with the problem of recovering a true signal from incomplete or noisy data. For example, weak signals from deep space probes are often so overwhelmed with background noise that the signal itself is barely detectable, yet the signal must be used to produce a photograph or provide other information. Researchers at Stanford University and elsewhere have been working for several years on using wavelet methods to filter out such noise. For example, Figure 2 shows a signal from a medical imaging signal that has been cleaned up (de-noised) using wavelets.

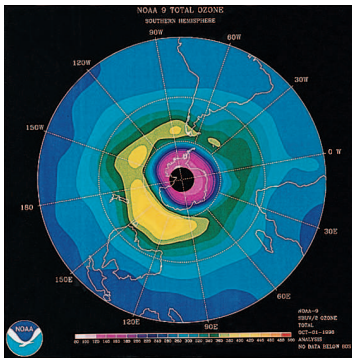


Noisy signal
De-noised signal
Figure 2



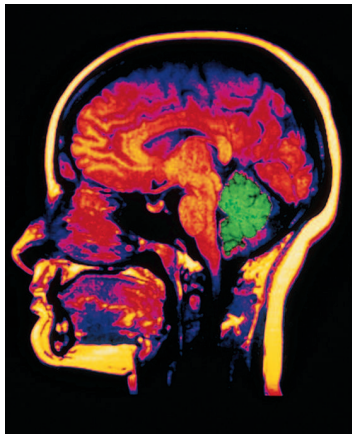
Airflow past a Saturn SL2

Figure 3



Ozone hole in the Southern Hemisphere

Figure 4



Magnetic resonance image

Figure 5



Chaotic ventricular fibrillation

Figure 6

Airflow Past an Automobile—Problems involving fluid flow (air, water, and blood, for example) are a major focus of scientific research. The Army High Performance Computing Research Center (AHPCRC) sponsors numerous unclassified research projects that involve teams of researchers from various science and engineering disciplines. One such project deals with airflow past an automobile (they use a General Motors Saturn SL2). The problem is quite complex since it takes into account the body contours, the wheels, the recessed headlights, and the spoiler. Figure 3 shows a simulation of airflow past an automobile that was produced using state-of-the-art mathematical methods and a Cray T3D supercomputer.

Weather Prediction—Modern meteorology is a marriage between mathematics and physics. Today's meteorologists are concerned with much more than predicting daily weather changes—their research delves into such areas as global warming, holes in the ozone layer (Figure 4), and weather patterns on other planets. In 1904 the Norwegian meteorologist Vilhelm Bjerknes (1862–1951) proposed that the state of the atmosphere at any future time can be determined by measuring appropriate variables at a single instant of time and then solving certain hydrodynamic equations. Although Bjerknes' idea is true in principle, it is difficult to apply because of uncertainties in measured variables, the enormous amounts of data to be processed, and technical complications involved with solving the equations. However, new mathematical discoveries have dramatically improved meteorological predictions and spawned enormous economic benefits. For example, it costs about 50 million dollars to prepare for a hurricane over 300 nautical miles of coastline, even if the hurricane does not hit the area. On the other hand, if the hurricane hits without adequate preparation, then the added costs can mount to billions of dollars (let alone the loss of life). Thus, each new mathematical breakthrough that produces more accurate hurricane prediction translates into enormous economic savings and preservation of human life.

Medical Imaging and DNA Structure—Advances in *nuclear magnetic resonance* (NMR) have made it possible to determine the structure of biological macromolecules, study DNA replication, and determine how proteins act as enzymes and antibodies. Related advances in *magnetic resonance imaging* (MRI) have made it possible to view internal human tissue without invasive surgery and to provide real-time images during surgical procedures (Figure 5). High-quality NMR and MRI would not be possible without mathematical discoveries that have occurred within the last decade.

Controlling Chaotic Behavior in the Human Heart—Chaos theory, which is one of the most exciting new branches of mathematics, is concerned with identifying regularities in phenomena that on the surface seem random and unpredictable (Figure 6). Today's research literature abounds with applications of chaos theory to almost every imaginable branch of science. Researchers at the Applied Chaos Laboratory at Georgia Tech University collaborated with physicians at the Emory University Medical Center in applying chaos theory to control the chaotic behavior of heart tissue that is undergoing ventricular fibrillation (cardiac arrest). The research, though experimental, is already showing promising results.

The World Model of the Future—In anticipation of the 1992 United Nations Earth Summit, researchers at the Institute for Economic Analysis (IEA) at New York University were commissioned by a number of world leaders with the daunting task of creating a model that would predict the economic and environmental future of the world. They started with the World Model and World Database developed by Nobel laureate Wassily Leontief and his colleagues at Harvard in the 1970s, but they expanded on the model by incorporating such environmental factors as the cost of controlling pollutant emissions (from mining, energy creation, and automobiles, for example). They also accounted for the effect of population growth rates on the added demand for energy and other natural resources. Models such as this require a team effort by government, academic, and industrial experts in a variety of fields and play an important role in guiding the decisions of governmental agencies.

Deep Space Exploration—Alexander Wolszczan of Penn State University may go down in history as the first scientist to identify a planetary system beyond our own. While

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searching the radio sky, Professor Wolszczan discovered a new pulsar, PSR1257+12, that seemed to wobble as it traveled through space. As a result of an extensive mathematical analysis, many scientists are now convinced that the wobble is caused by two or three planets orbiting PSR1257+12. Although scientists have been able to detect pulsars for some time by searching for faint periodic radio signals from outer space, it is only recently that the mathematical techniques have been developed to analyze the data in a way that stands up to scientific scrutiny. Wolszczan predicts that the planets orbiting PSR1257+12 are barren and inhospitable because of stellar winds, but his methods open the possibility of discovering new planetary systems that may sustain intelligent life.

THE ROOTS OF CALCULUS

Today's exciting applications of calculus have roots that can be traced to the work of the Greek mathematician Archimedes, but the actual discovery of the fundamental principles of calculus was made independently by Isaac Newton (English) and Gottfried Leibniz (German) in the late seventeenth century. The work of Newton and Leibniz was motivated by four major classes of scientific and mathematical problems of the time:

- Find the tangent line to a general curve at a given point.
- Find the area of a general region, the length of a general curve, and the volume of a general solid.
- Find the maximum or minimum value of a quantity—for example, the maximum and minimum distances of a planet from the Sun, or the maximum range attainable for a projectile by varying its angle of fire.
- Given a formula for the distance traveled by a body in any specified amount of time, find the velocity and acceleration of the body at any instant. Conversely, given a formula that specifies the acceleration of velocity at any instant, find the distance traveled by the body in a specified period of time.

Newton and Leibniz found a fundamental relationship between the problem of finding a tangent line to a curve and the problem of determining the area of a region. Their realization of this connection is considered to be the “discovery of calculus.”

Though Newton saw how these two problems are related 10 years before Leibniz, Leibniz published his work 20 years before Newton. This situation led to a story debate over which was the rightful discoverer of calculus. The debate engulfed Europe for half a century, with the scientists of the European continent supporting Leibniz and those from England supporting Newton. The conflict was extremely unfortunate because Newton's inferior notation badly hampered scientific development in England, and the Continent in turn lost the benefit of Newton's discoveries in astronomy and physics for nearly 50 years. In spite of it all, Newton and Leibniz were sincere admirers of each other's work.



ISAAC NEWTON (1642–1727)

Newton was born in the village of Woolsthorpe, England. His father died before he was born and his mother raised him on the family farm. As a youth he showed little evidence of his later brilliance, except for an unusual talent with mechanical devices—he apparently built a working water clock and a toy flour mill powered by a mouse. In 1661 he entered Trinity College in Cambridge with a deficiency in geometry. Fortunately, Newton caught the eye of Isaac Barrow, a gifted mathematician and teacher. Under Barrow's guidance Newton immersed himself in mathematics and science, but he graduated without any special distinction. Because the Plague was spreading rapidly through London, Newton returned to his home in Woolsthorpe and stayed there during the years of 1665 and 1666. In those two momentous years the entire framework of modern science was miraculously created in Newton's mind—he discovered calculus, recognized the underlying principles of planetary

motion and gravity, and determined that “white” sunlight was composed of all colors, red to violet. For whatever reasons he kept his discoveries to himself. In 1667 he returned to Cambridge to obtain his Master’s degree and upon graduation became a teacher at Trinity. Then in 1669 Newton succeeded his teacher, Isaac Barrow, to the Lucasian chair of mathematics at Trinity, one of the most honored chairs of mathematics in the world. Thereafter, brilliant discoveries flowed from Newton steadily. He formulated the law of gravitation and used it to explain the motion of the Moon, the planets, and the tides; he formulated basic theories of light, thermodynamics, and hydrodynamics; and he devised and constructed the first modern reflecting telescope.

Throughout his life Newton was hesitant to publish his major discoveries, revealing them only to a select circle of friends, perhaps because of a fear of criticism or controversy. In 1687, only after intense coaxing by the astronomer, Edmond Halley (Halley’s comet), did Newton publish his masterpiece, *Philosophiae Naturalis Principia Mathematica* (The Mathematical Principles of Natural Philosophy). This work is generally considered to be the most important and influential scientific book ever written. In it Newton explained the workings of the solar system and formulated the basic laws of motion, which to this day are fundamental in engineering and physics. However, not even the pleas of his friends could convince Newton to publish his discovery of calculus. Only after Leibniz published his results did Newton relent and publish his own work on calculus.

After 25 years as a professor, Newton suffered depression and a nervous breakdown. He gave up research in 1695 to accept a position as warden and later master of the London mint. During the 25 years that he worked at the mint, he did virtually no scientific or mathematical work. He was knighted in 1705 and on his death was buried in Westminster Abbey with all the honors his country could bestow. It is interesting to note that Newton was a learned theologian who viewed the primary value of his work to be its support of the existence of God. Throughout his life he worked passionately to date biblical events by relating them to astronomical phenomena. He was so consumed with this passion that he spent years searching the Book of Daniel for clues to the end of the world and the geography of hell.

Newton described his brilliant accomplishments as follows: “I seem to have been only like a boy playing on the seashore and diverting myself in now and then finding a smoother pebble or prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me.”

GOTTFRIED WILHELM LEIBNIZ (1646–1716)

This gifted genius was one of the last people to have mastered most major fields of knowledge—an impossible accomplishment in our own era of specialization. He was an expert in law, religion, philosophy, literature, politics, geology, metaphysics, alchemy, history, and mathematics.

Leibniz was born in Leipzig, Germany. His father, a professor of moral philosophy at the University of Leipzig, died when Leibniz was six years old. The precocious boy then gained access to his father’s library and began reading voraciously on a wide range of subjects, a habit that he maintained throughout his life. At age 15 he entered the University of Leipzig as a law student and by the age of 20 received a doctorate from the University of Altdorf. Subsequently, Leibniz followed a career in law and international politics, serving as counsel to kings and princes.

During his numerous foreign missions, Leibniz came in contact with outstanding mathematicians and scientists who stimulated his interest in mathematics—most notably, the physicist Christian Huygens. In mathematics Leibniz was self-taught, learning the subject by reading papers and journals. As a result of this fragmented mathematical education, Leibniz often rediscovered the results of others, and this helped to fuel the debate over the discovery of calculus.



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Leibniz never married. He was moderate in his habits, quick-tempered, but easily appeased, and charitable in his judgment of other people's work. In spite of his great achievements, Leibniz never received the honors showered on Newton, and he spent his final years as a lonely embittered man. At his funeral there was one mourner, his secretary. An eyewitness stated, "He was buried more like a robber than what he really was—an ornament of his country."