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a) $f(x,y,z) = xyz$ $\quad x^2+y^2+z^2 = 1$

$g(x,y,z) = x^2+y^2+z^2-1$

1)
$$\begin{cases} \frac{\partial f}{\partial x} = \lambda \frac{\partial g}{\partial x} \\ \frac{\partial f}{\partial y} = \lambda \frac{\partial g}{\partial y} \\ \frac{\partial f}{\partial z} = \lambda \frac{\partial g}{\partial z} \\ x^2+y^2+z^2 = 1 \end{cases} \Rightarrow \begin{cases} yz = \lambda 2x \\ xz = \lambda 2y \\ xy = \lambda 2z \end{cases} \Rightarrow \begin{cases} \lambda = \frac{yz}{2x} \\ \lambda = \frac{xz}{2y} \\ \lambda = \frac{xy}{2z} \end{cases} \Rightarrow \begin{cases} \underbrace{\frac{yz}{2x} = \frac{xz}{2y}}_{1} = \underbrace{\frac{xy}{2z}}_{2} \\ = \\ = \end{cases}$$

$(0,0,0)$ não é pt. crítico

$$\begin{cases} \frac{yz}{2x} = \frac{xz}{2y} \\ \frac{xz}{2y} = \frac{xy}{2z} \\ \frac{yz}{2x} = \frac{xy}{2z} \end{cases} \Rightarrow \begin{cases} y^2z = x^2z \\ xz^2 = xy^2 \\ yz^2 = x^2y \end{cases} \Rightarrow \begin{cases} y = \pm x \\ z = \pm y \\ z = \pm x \end{cases}$$

$x^2+x^2+x^2 = 1 \Leftrightarrow x^2 = \frac{1}{3} \quad x = \pm \frac{1}{\sqrt{3}}$

$f(0,0,0)$ não é ponto crítico

$\underbrace{\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)}_{\max}$, $\underbrace{\left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)}_{\max}$, $\underbrace{\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)}_{\min}$,

$\underbrace{\left(\frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)}_{\max}$ $\underbrace{\left(-\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)}_{\min}$ $\underbrace{\left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)}_{\min}$,

$\underbrace{\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)}_{\min}$, $\underbrace{\left(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)}_{\max}$

$f\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right) = \frac{1}{3\sqrt{3}}$ $\quad f\left(-\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right) = \frac{-1}{3\sqrt{3}}$

min