

Expressão	Substituição
$f(x) = R(x^{\frac{m}{n}}, x^{\frac{p}{q}}, \dots, x^{\frac{r}{s}})$	$x = t^\mu$ $\mu = \text{m.m.c.}\{n, q, \dots, s\}$
$f(x) = R\left(x, \left(\frac{ax+b}{cx+d}\right)^{\frac{m}{n}}, \left(\frac{ax+b}{cx+d}\right)^{\frac{p}{q}}, \dots, \left(\frac{ax+b}{cx+d}\right)^{\frac{r}{s}}\right)$	$\frac{ax+b}{cx+d} = t^\mu$ $\mu = \text{m.m.c.}\{n, q, \dots, s\}$
$f(x) = x^\alpha (a + b x^\beta)^\gamma$	$x^\beta = t$

Expressão	Substituição
	$\sqrt{a x^2 + b x + c} = \sqrt{a} x + t$ se $a > 0$
$f(x) = R(x, \sqrt{a x^2 + b x + c})$	$\sqrt{a x^2 + b x + c} = t x + \sqrt{c}$ se $c > 0$

$\sqrt{a x^2 + b x + c} = t (x - \alpha)$
ou $\sqrt{a x^2 + b x + c} = t (x - \beta)$
se α e β são zeros reais
distintos de $a x^2 + b x + c$

Expressão	Substituição
$\sqrt{a^2 - x^2}$	$x = a \cos(t)$ ou $x = a \sin(t)$
$\sqrt{x^2 - a^2}$	$x = a \sec(t)$ ou $x = a \cosec(t)$
$\sqrt{x^2 + a^2}$	$x = a \tg(t)$ ou $x = a \cotg(t)$

Expressão	Substituição
$f(x) = R(\sin(x), \cos(x))$	$\operatorname{tg}\left(\frac{x}{2}\right) = t$
$f(x) = R(\sin(x), \cos(x))$ $R(-y, -z) = R(y, z), \forall y, z$	$\operatorname{tg}(x) = t$
$f(x) = R(e^x)$	$e^x = t$

$$\operatorname{tg}\left(\frac{x}{2}\right) = t \Rightarrow x = 2 \operatorname{arctg}(t) = \varphi(t) \Rightarrow \varphi'(t) = \frac{2}{1+t^2},$$

$$\begin{aligned} \cos(x) &= \cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right) = \frac{1}{1+\operatorname{tg}^2\left(\frac{x}{2}\right)} - \frac{\operatorname{tg}^2\left(\frac{x}{2}\right)}{1+\operatorname{tg}^2\left(\frac{x}{2}\right)} \\ &= \frac{1-\operatorname{tg}^2\left(\frac{x}{2}\right)}{1+\operatorname{tg}^2\left(\frac{x}{2}\right)} = \frac{1-t^2}{1+t^2} \end{aligned}$$

$$\begin{aligned} \sin(x) &= 2 \sin\left(\frac{x}{2}\right) \cdot \cos\left(\frac{x}{2}\right) = 2 \frac{\operatorname{tg}\left(\frac{x}{2}\right)}{\sqrt{1+\operatorname{tg}^2\left(\frac{x}{2}\right)}} \cdot \frac{1}{\sqrt{1+\operatorname{tg}^2\left(\frac{x}{2}\right)}} \\ &= 2 \frac{\operatorname{tg}\left(\frac{x}{2}\right)}{1+\operatorname{tg}^2\left(\frac{x}{2}\right)} = \frac{2t}{1+t^2} \end{aligned}$$

$$\varphi(t) = \operatorname{arctg}(t)$$

$$\sin(x) = \frac{t}{\sqrt{1+t^2}} \quad \text{e} \quad \cos(x) = \frac{1}{\sqrt{1+t^2}}.$$