1.

a) $f'(x) = \left(x e^{-x^2}\right)' = e^{-x^2} + x(-2x)e^{-x^2}$

$= e^{-x^2}(1-2x^2)$

$f'(0) = e^0(1-0) = 1$

b) $f'(x) = \left(\sqrt[3]{1+\operatorname{arctg}(x)}\right)' = \frac{1}{3} \times \frac{(1+\operatorname{arctg}(x))'}{\sqrt[3]{(1+\operatorname{arctg}(x))^2}}$

$= \frac{1}{3} \times \frac{\frac{1}{1+x^2}}{\sqrt[3]{(1+\operatorname{arctg}(x))^2}}$

$= \frac{1}{(3+3x^2)\sqrt[3]{(1+\operatorname{arctg}(x))^2}}$

$f'\left(\frac{\pi}{4}\right) = \frac{1}{\left(3+3\times\left(\frac{\pi}{4}\right)^2\right)\sqrt[3]{\left(1+\operatorname{arctg}\left(\frac{\pi}{4}\right)\right)^2}}$

$= \frac{1}{\left(3+3\times\frac{\pi^2}{16}\right)\sqrt[3]{\left(1+\operatorname{arctg}\left(\frac{\pi}{4}\right)\right)^2}}$

c) $f'(x) = \begin{cases} (1-\log(x^3+2x))' & \text{se } x>0 \\ \left(x+\frac{1}{x+1}\right)' & \text{se } x\leq 0 \end{cases}$

$= \begin{cases} -\frac{3x^2+2}{x^3+2x} & \text{se } x>0 \\ 1-\frac{1}{(x+1)^2} & \text{se } x\leq 0 \end{cases}$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x+\frac{1}{x+1} = 1$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-}(1-\log(x^3+2x)) = +\infty$

Como $\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$, f não é contínua em $x=0$

$f'(0)$ não existe

$f'(1) = -\frac{3+2}{1+2} = -\frac{5}{3}$

$f'(-2) = 1-\frac{1}{(-2+1)^2} = 0$

d) $f'(x) = \begin{cases} \left(\operatorname{arcsen}\left(\frac{1}{x}\right)-\frac{\pi}{2}\right)' & \text{se } x>1 \\ \left(\frac{x^2+x-2}{x+3}\right)' & \text{se } x\leq 1 \end{cases}$

$= \begin{cases} \frac{1}{\sqrt{1-\frac{1}{x^2}}\,x^2} & \text{se } x>1 \\ \frac{(2x+1)(x+3)-x^2-x+2}{(x+3)^2} & \text{se } x\leq 1 \end{cases}$

$= \begin{cases} -\frac{1}{x^2\sqrt{1-\frac{1}{x^2}}} & \text{se } x>1 \\ \frac{x^2+6x+5}{(x+3)^2} & \text{se } x\leq 1 \end{cases}$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \frac{x^2+x-2}{x+3} = \frac{0}{4} = 0$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}\left(\operatorname{arcsen}\left(\frac{1}{x}\right)-\frac{\pi}{2}\right) = \frac{\pi}{2}-\frac{\pi}{2} = 0$

Como $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$ então

f é contínua em todo o seu domínio

$f'(1) = \frac{1+6+5}{16} = \frac{12}{16} = \frac{6}{8} = \frac{3}{4}$

$f'(2) = -\frac{1}{4\sqrt{1-\frac{1}{4}}} = -\frac{1}{4\sqrt{\frac{3}{4}}} = -\frac{1}{4\frac{\sqrt{3}}{2}} = -\frac{1}{2\sqrt{3}}$

(2)

a) $f'(x) = \left(\log(\log(x))\right)' = \dfrac{(\log(x))'}{\log(x)} = \dfrac{1}{x\log(x)}$

$f'(e) = \dfrac{1}{e \times \log(e)} = \dfrac{1}{e}$ $\qquad f(e) = \log(\log(e)) = 0$

$y = f'(e)(x-e) + f(e) \Leftrightarrow y = \dfrac{1}{e}(x-e) + 0$

$\Leftrightarrow y = \dfrac{1}{e}x - 1$

b) Quando $x=-1$, ou seja, $x<0$:

$f'(x) = \left(-e^{\frac{1}{x}}\right)' = \dfrac{e^{\frac{1}{x}}}{x^2}$ $\qquad f'(-1) = \dfrac{e^{-1}}{1} = \dfrac{1}{e}$

$f(-1) = -e^{-1} = -\dfrac{1}{e}$

$y = f'(-1)(x+1) + f(-1) \Leftrightarrow y = \dfrac{1}{e}(x+1) - \dfrac{1}{e}$

$\Leftrightarrow y = \dfrac{1}{e}x$

c) Quando $x=0$:

$f'(x) = (0)' = 0$ $\qquad f'(0) = 0$ $\qquad y = f'(0)(x-0) + f(0)$

$f(0) = 0$ $\qquad \Leftrightarrow y = 0$

(3)

a)

$$f'(x) = \begin{cases} \left(\frac{\pi}{2} + \log(1-x^2)\right)' & \text{se } x \leq 0 \\ \left(\operatorname{arctg}\left(\frac{1}{x}\right) + \frac{1}{2}x\right)' & \text{se } x > 0 \end{cases}$$

$$= \begin{cases} \dfrac{-2x}{1-x^2} & \text{se } x \leq 0 \\ \dfrac{-\frac{1}{x^2}}{1+\frac{1}{x^2}} + \dfrac{1}{2} & \text{se } x > 0 \end{cases} = \begin{cases} \dfrac{2x}{x^2-1} & \text{se } x \leq 0 \\ -\dfrac{1}{x^2+1} + \dfrac{1}{2} & \text{se } x > 0 \end{cases}$$

$$= \begin{cases} \dfrac{2x}{x^2-1} & \text{se } x \leq 0 \\ \dfrac{x^2-1}{2x^2+2} & \text{se } x > 0 \end{cases}$$

b)

$$f'(x) = \begin{cases} (-\pi - 1 - x)' & \text{se } x \leq -\pi \\ (\cos(x))' & \text{se } -\pi < x < 0 \\ \left(\frac{1}{1+x^2}\right)' & \text{se } x \geq 0 \end{cases}$$

$$= \begin{cases} -1 & \text{se } x \leq -\pi \\ -\operatorname{sen}(x) & \text{se } -\pi < x < 0 \\ \frac{-2x}{(x^2+1)^2} & \text{se } x \geq 0 \end{cases}$$

c)

$$f'(x) = \begin{cases} (\operatorname{arcsen}(x^2-1))' & \text{se } x < 1 \\ \left(\frac{3}{x} - 3x^3\right)' & \text{se } 1 \leq x \leq 5 \end{cases}$$

$$= \begin{cases} \frac{2x}{\sqrt{1-(x^2-1)^2}} & \text{se } x < 1 \\ -\frac{3}{x^2} - 9x^2 & \text{se } 1 \leq x \leq 5 \end{cases} = \begin{cases} \frac{2x}{\sqrt{1-(x^2-1)^2}} & \text{se } x < 1 \\ -\frac{9x^4+3}{x^2} & \text{se } 1 \leq x \leq 5 \end{cases}$$

d)

$$f'(x) = \begin{cases} (-x^2 e^{1-x})' & \text{se } x \geq 0 \\ \frac{x^2}{x^2-4} & \text{se } x < 0 \end{cases}$$

$$= \begin{cases} -2x e^{1-x} + (-e^{1-x})(-x^2) & \text{se } x \geq 0 \\ \frac{2x(x^2-4) - 2x(x^2)}{(x^2-4)^2} & \text{se } x < 0 \end{cases}$$

$$= \begin{cases} e^{1-x}(x^2-2x) & \text{se } x \geq 0 \\ \frac{-8x}{(x^2-4)^2} & \text{se } x < 0 \end{cases}$$

e)

$$f'(x) = \begin{cases} \left(\operatorname{arcsen}\left(\frac{1}{x}\right) - \frac{\pi}{2}\right)' & \text{se } x > 1 \\ \frac{x^2+x-2}{x+3} & \text{se } x \leq 1 \end{cases}$$

$$= \begin{cases} \frac{-\frac{1}{x^2}}{\sqrt{1-\frac{1}{x^2}}} & \text{se } x > 1 \\ \frac{(2x+1)(x+3) - x^2 - x + 2}{(x+3)^2} & \text{se } x \leq 1 \end{cases}$$

$$= \begin{cases} -\frac{1}{x^2\sqrt{1-\frac{1}{x^2}}} & \text{se } x > 1 \\ \frac{x^2+6x+5}{(x+3)^2} & \text{se } x \leq 1 \end{cases}$$

f) $f'(x) = \begin{cases} \left(\dfrac{x^2}{x^2-1}\right)' & \text{se } x \geq -2 \\ \left(e^{|x+3|}\right)' & \text{se } x < -2 \end{cases}$

(i) $|x+3| = \begin{cases} -x-3 & \text{se } x < -3 \\ x+3 & \text{se } x \geq -3 \end{cases}$

$\overset{(i)}{=} \begin{cases} \dfrac{2x(x^2-1) - 2x(x^2)}{(x^2-1)^2} & \text{se } x \geq -2 \\ e^{-x-3} & \text{se } x < -2 \text{ e } x < -3 \\ e^{x+3} & \text{se } x < -2 \text{ e } x \geq -3 \end{cases}$

$= \begin{cases} -\dfrac{2x}{(x^2-1)^2} & \text{se } x \geq -2 \\ e^{-x-3} & \text{se } x < -3 \\ e^{x+3} & \text{se } -3 \leq x < -2 \end{cases}$

4. $g'(x) = (f(\operatorname{arctg}(x)))'$

$= f'(\operatorname{arctg}(x)) \times (\operatorname{arctg}(x))'$

$= f'(\operatorname{arctg}(x)) \times \dfrac{1}{1+x^2}$

$g''(x) = \left(f'(\operatorname{arctg}(x)) \times \dfrac{1}{1+x^2}\right)'$

$= \dfrac{(f'(\operatorname{arctg}(x)))'(1+x^2) - 2x(f'(\operatorname{arctg}(x))}{(x^2+1)^2}$

$= \dfrac{f''(\operatorname{arctg}(x)) \times \cancel{\frac{1}{1+x^2}} \times (\cancel{1+x^2}) - 2x(f'(\operatorname{arctg}(x))}{(x^2+1)^2}$

$4g''(1) = \cancel{4} \times \dfrac{f''(\operatorname{arctg}(1)) - 2f'(\operatorname{arctg}(1))}{\cancel{4}} = f''\left(\frac{\pi}{4}\right) - 2f'\left(\frac{\pi}{4}\right)$ cqd.

5.

a) $g'(x) = (f(e^x))' = f'(e^x) \times (e^x)' = f'(e^x)\, e^x$

b) $g''(x) = (g'(x))' = (f'(e^x)\, e^x)' = f''(e^x) \times e^{2x} + e^x f'(e^x)$

$= e^x(f''(e^x)\, e^x + f'(e^x))$

$g''(1) = e(f''(e)\, e + f'(e))$

6. $h'(x) = ((g \circ f)(\operatorname{sen}(x)))' = g'(f(\operatorname{sen}(x))) \times f'(\operatorname{sen}(x))$

(i) $f'(x) = 4x^3$

$\overset{(i)}{=} g'(\operatorname{sen}^4(x)) \times 4 \operatorname{sen}^3(x)$

7. $((f \circ g)^{-1}(x))' = \dfrac{1}{(f(g(x)))'} = \dfrac{1}{f'(g(x)) \times g'(x)} =$

$= \dfrac{1}{f'(x^3) \times 3x^2}$