Digital systems: from bits to microcontrollers

- Introduction to digital systems fundamentals
- Combinatorial digital systems
- Sequential digital systems
- Introduction to microcontrollers
- Introduction to advanced implementation platforms

Introduction to digital systems fundamentals

Bibliography:

Digital Logic Circuit Analysis & Design - Victor P. Nelson, H. Troy Nagle, J. David Irwin, Bill D. Carroll - Prentice Hall - ISBN 0-13-463894-8

In Portuguese:

Circuitos Digitais e Microprocessadores - Herbert Taub - McGraw-Hill - ISBN 0-07-066595-8

Alternatives:

Logic and Computer Design Fundamentals - M. Morris Mand, Charles Kime - Prentice-Hall - ISBN 0-13-182098-2

Digital Design - Principles and Practice - John F. Wakerly - Prentice-Hall - ISBN 0-13-082599-9

Measurement of the level of the water in a tank

Comments / suggestions ?

Measurement of the level of the water in a tank - I

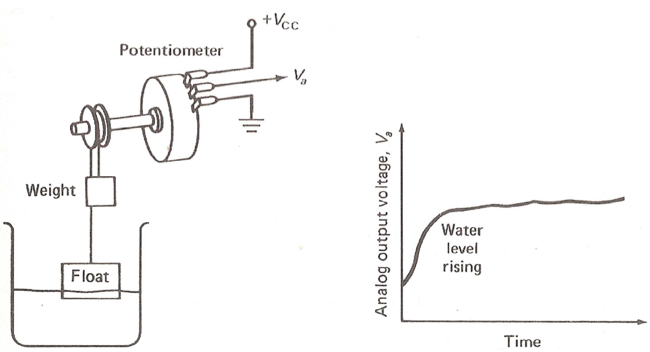
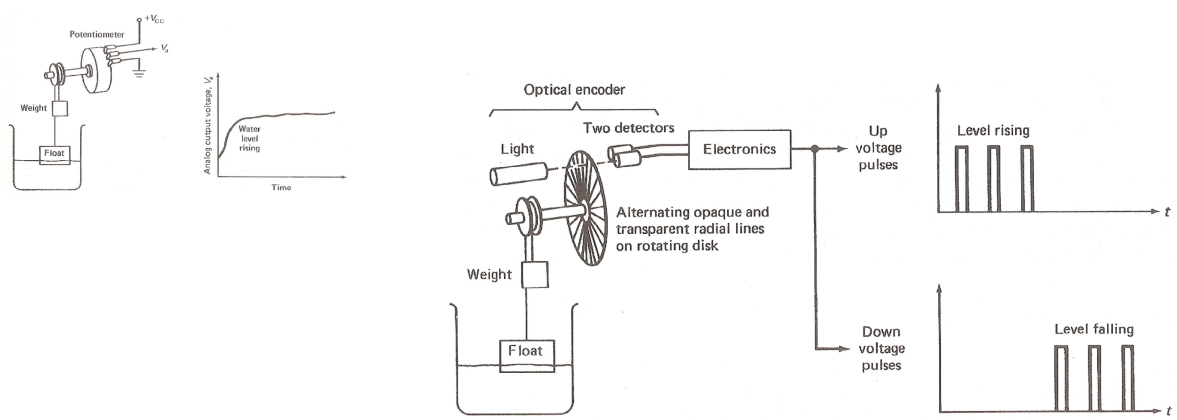


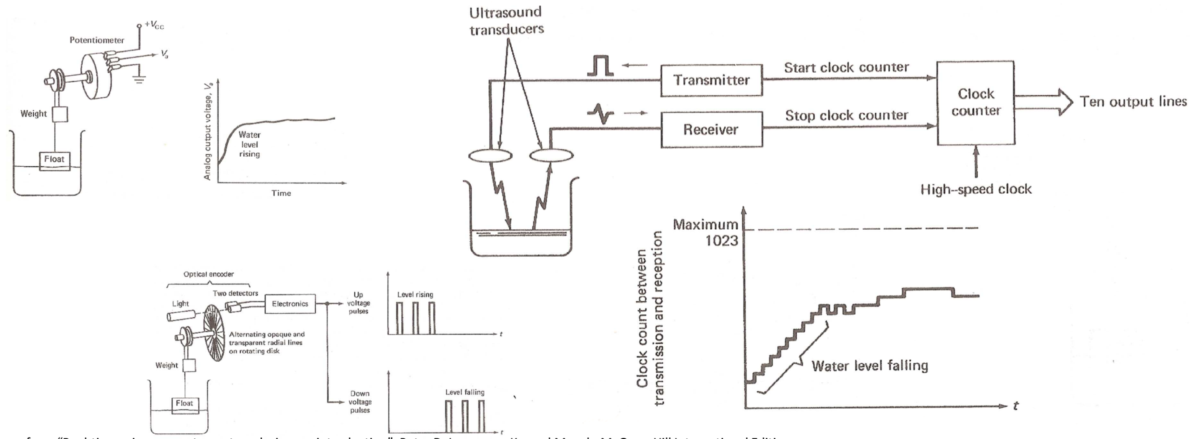
Figure from "Real-time microcomputer system design: an introduction", Peter D. Lawrence, Konrad Mauch; McGraw-Hill International Editions

Measurement of the level of the water in a tank - II



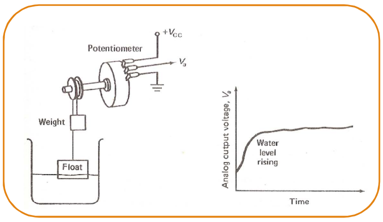
Figures from “Real-time microcomputer system design: an introduction”, Peter D. Lawrence, Konrad Mauch; McGraw-Hill International Editions

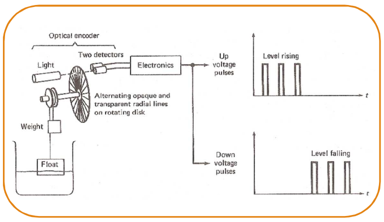
Measurement of the level of the water in a tank - III

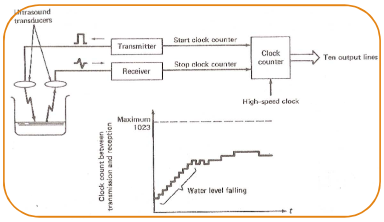


Figures from “Real-time microcomputer system design: an introduction”, Peter D. Lawrence, Konrad Mauch; McGraw-Hill International Editions

Measurement of the level of the water in a tank: different types of solutions





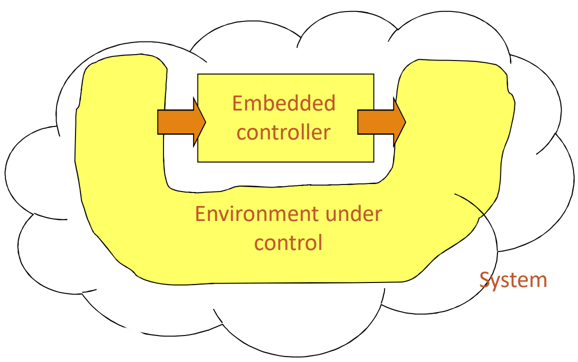


Analog versus digital

Mechanical versus solid-state

Figures from “Real-time microcomputer system design: an introduction”, Peter D. Lawrence, Konrad Mauch; McGraw-Hill International Editions

Embedded controller and environment under control



Architecture of an industrial control system

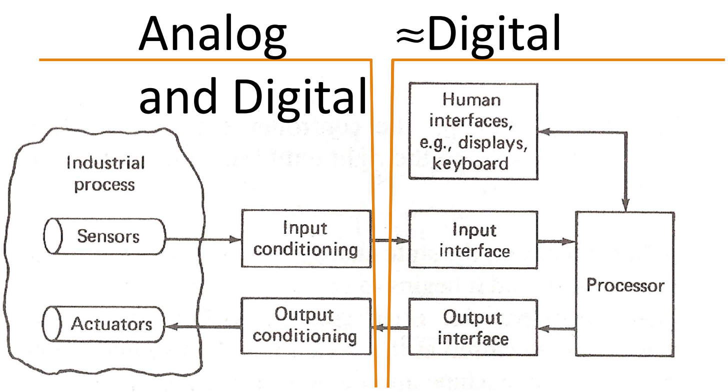
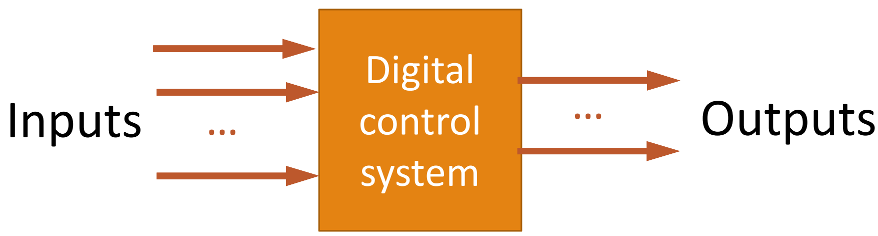
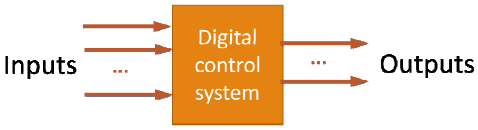


Figure adapted from "Real-time microcomputer system design: an introduction", Peter D. Lawrence, Konrad Mauch; McGraw-Hill International Editions

Black box characterization of a digital system



Characterization of a digital system



- When outputs depend on inputs at one specific instant

Combinatorial circuits

- When outputs depend on the evolution over time of the inputs

Sequential circuits

Boolean algebra (1849)

Boolean variables can hold two truth values:

- True, False
- 1,0
- ON, OFF

Two basic operations are defined:

- OR (represented as +)
- AND (represented as .)   complementing the Negation (NOT)

Boolean algebra defined over:

{U = { 0 , 1 }, +, . }

LUÍS GOMES, 2014,15

Basic postulates involving one variable

$A + 0 = A$ (neutral) $A + 1 = 1$ (null element)

$A \cdot 1 = A$ $A \cdot 0 = 0$

Duality principle (+←→. ; 0←→1)

$A + A = A$ $A + \bar{A} = 1$

$A \cdot A = A$ $A \cdot \bar{A} = 0$

$A = \bar{\bar{A}}$ (involution)

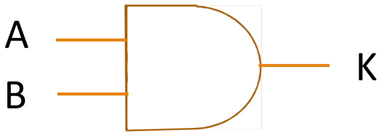
Introducing AND function

$K = A \cdot B$

Expression

A	B	K
0	0	0
0	1	0
1	0	0
1	1	1

Truth table

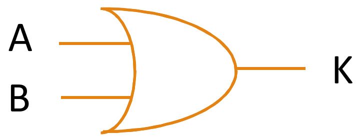


Schematic

Introducing OR function

$K = A + B$

A	B	K
0	0	0
0	1	1
1	0	1
1	1	1

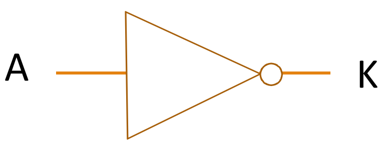


Expression Truth table Schematic

Introducing NOT function

$K = \overline{A}$

A	K
0	1
1	0

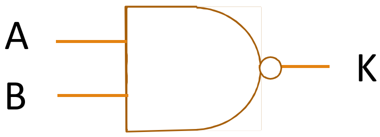


Expression Truth table Schematic

Introducing NAND function

$K = \overline{A . B}$

A	B	K
0	0	1
0	1	1
1	0	1
1	1	0

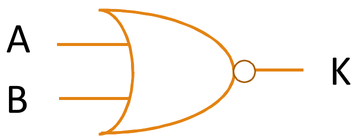


Expression Truth table Schematic

Introducing NOR function

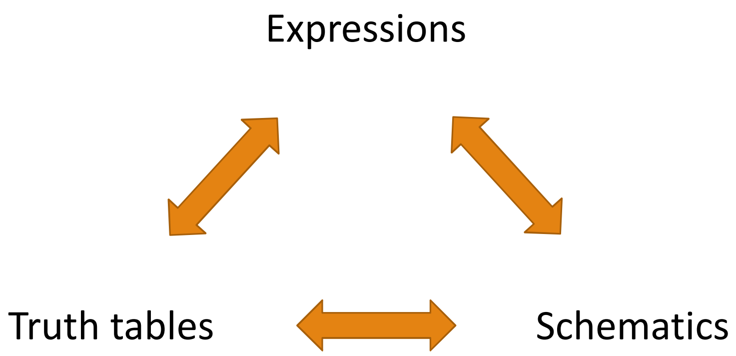
$K = \overline{A + B}$

A	B	K
0	0	1
0	1	0
1	0	0
1	1	0



Expression Truth table Schematic

Equivalent representations



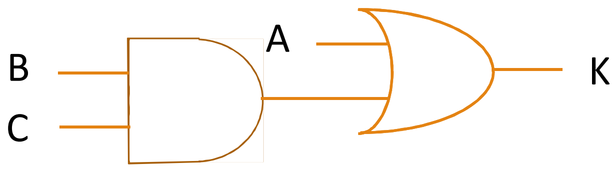
Describing a function

K = A + B.C

Expression

A	B	C	K
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Truth table



Schematic

Sixteen functions of two variables

A	B	f_0	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}	f_{12}	f_{13}	f_{14}	f_{15}
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

Sixteen functions of two variables

NAND

NOR

XOR

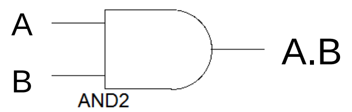
XNOR

A	B	f_0	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}	f_{12}	f_{13}	f_{14}	f_{15}
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

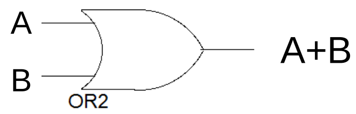
INV

AND

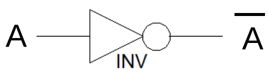
OR



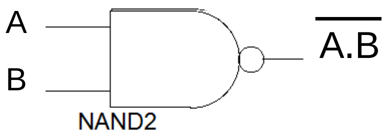
A	B	A.B
0	0	0
0	1	0
1	0	0
1	1	1



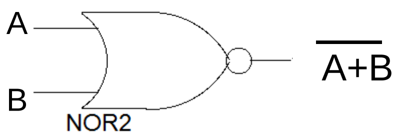
A	B	A+B
0	0	0
0	1	1
1	0	1
1	1	1



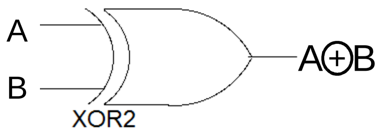
A	$\overline{A}$
0	1
1	0



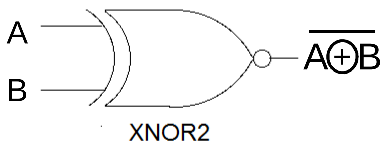
A	B	A.B	$\overline{A.B}$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0



A	B	A+B	$\overline{A+B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

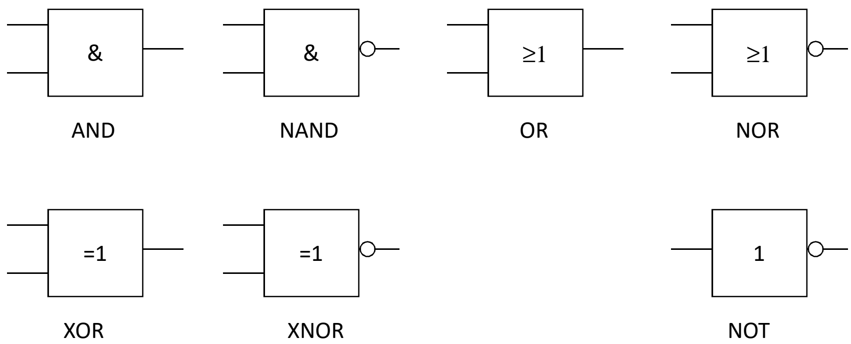


A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

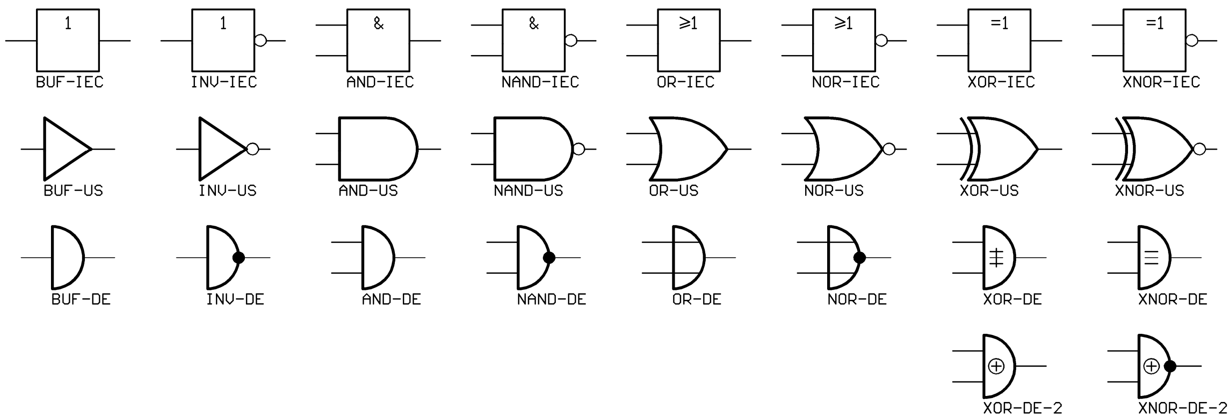


A	B	$\overline{A \oplus B}$
0	0	1
0	1	0
1	0	0
1	1	1

Standart schematic symbols



More on schematic symbols



Basic postulates and theorems

A + B = B + A   A . B = B . A   (commutative)

A + B + C = (A + B) + C = A + (B + C)   (associative)

A . B . C = (A . B) . C = A . (B . C)

A . (B + C) = A . B + A . C   (distributive)

A + B . C = (A + B) . (A + C)

A + A.B = A   A . (A+B) = A   (absortion)

DeMorgan's theorems

$\overline{A . B} = \overline{A} + \overline{B}$ $\overline{A + B} = \overline{A} . \overline{B}$

More basic theorems

$A + \overline{A}.B = A + B$ — $A . (\overline{A} + B) = A.B$

$A.B + A.\overline{B} = A$ — $(A + B).(A + \overline{B}) = A$

$A.B + A.\overline{B}.C = A.B + A.C$ — $(A+B).(A+\overline{B}+C) = (A+B) . (A+C)$

$A.B + \overline{A}.C + B.C = A.B + \overline{A}.C$ (consensus)

$(A+B).(\overline{A}+C).(B+C) = (A+B).(\overline{A}+C)$

Some postulates and theorems

DeMorgan's laws

$\overline{A . B} = \overline{A} + \overline{B}$ — $\overline{A + B} = \overline{A} . \overline{B}$

$A = \overline{\overline{A}}$ (involution)

$A + A = A$ — $A . A = A$

$A + 0 = A$ (neutral) — $A . 1 = A$

(1) $\overline{A . B} = \overline{A} + \overline{B}$

(2) $\overline{A + B} = \overline{A} . \overline{B}$

(3) $A = \overline{\overline{A}}$

(4) $A + A = A$

(5) $A . A = A$

(6) $A + 0 = A$

(7) $A . 1 = A$

OR+NOT or AND+NOT are sufficient

$$A.B = \overline{\overline{A.B}} = \overline{\overline{A} + \overline{B}}$$

(3) (1)

$$A+B = \overline{\overline{A+B}} = \overline{\overline{A} . \overline{B}}$$

(3) (2)

(1) $\overline{A . B} = \overline{A} + \overline{B}$

(2) $\overline{A + B} = \overline{A} . \overline{B}$

(3) $A = \overline{\overline{A}}$

(4) $A + A = A$

(5) $A . A = A$

(6) $A + 0 = A$

(7) $A . 1 = A$

NORs are sufficient

$$A.B = \overline{\overline{A.B}} = \overline{\overline{A} + \overline{B}} = \overline{\overline{A+A} + \overline{B+B}}$$

(3) (1) (4)

$$A+B = \overline{\overline{A+B}} \overset{(6)}{=} \overline{\overline{A+B} + 0}$$

$$A+B = \overline{\overline{A+B}} \overset{(4)}{=} \overline{\overline{A+B} + \overline{A+B}}$$

(3)

$$\overline{A} = \overline{A+A}$$

(4)

(1) $\overline{A . B} = \overline{A} + \overline{B}$

(2) $\overline{A + B} = \overline{A} . \overline{B}$

(3) $A = \overline{\overline{A}}$

(4) $A + A = A$

(5) $A . A = A$

(6) $A + 0 = A$

(7) $A . 1 = A$

NANDs are sufficient

$$A+B = \overline{\overline{A+B}} = \overline{\overline{A}\,.\,\overline{B}} = \overline{\overline{A.A}\,.\,\overline{B.B}}$$

3 2 5

$$A.B = \overline{\overline{A.B}} = \overline{\overline{A.B}\,.\,1} \quad (7)$$

$$A.B = \overline{\overline{A.B}} = \overline{\overline{A.B}\,.\,\overline{A.B}} \quad (5)$$

3

$$\overline{A} = \overline{A.A}$$

5

(1) $\overline{A\,.\,B} = \overline{A} + \overline{B}$

(2) $\overline{A + B} = \overline{A}\,.\,\overline{B}$

(3) $A = \overline{\overline{A}}$

(4) $A + A = A$

(5) $A\,.\,A = A$

(6) $A + 0 = A$

(7) $A\,.\,1 = A$

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Using only NANDs

$$A.B.C + A.D = \overline{\overline{A.B.C + A.D}} = \overline{\overline{A.B.C}\,.\,\overline{A.D}}$$

3 2

(1) $\overline{A\,.\,B} = \overline{A} + \overline{B}$

(2) $\overline{A + B} = \overline{A}\,.\,\overline{B}$

(3) $A = \overline{\overline{A}}$

(4) $A + A = A$

(5) $A\,.\,A = A$

(6) $A + 0 = A$

(7) $A\,.\,1 = A$

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LUÍS GOMES, 2014,15

Using only 2-input NANDs

$$A.B.C + A.D = \overline{\overline{A.B.C + A.D}} = \overline{\overline{A.B.C} \,.\, \overline{A.D}} =$$

3 2

$$= \overline{\overline{\overline{\overline{A.B}}.C} \,.\, \overline{A.D}} = \overline{\overline{\overline{\overline{A.B} \,.\, \overline{A.B}} \,.\, C} \,.\, \overline{A.D}}$$

3 5

(1) $\overline{A \,.\, B} = \overline{A} + \overline{B}$

(2) $\overline{A + B} = \overline{A} \,.\, \overline{B}$

(3) $A = \overline{\overline{A}}$

(4) $A + A = A$

(5) $A \,.\, A = A$

(6) $A + 0 = A$

(7) $A \,.\, 1 = A$

Using only NORs

$$A.B.C + A.D = \overline{\overline{\overline{\overline{A.B.C}} + \overline{\overline{A.D}}}} = \overline{\overline{\overline{\overline{A}+\overline{B}+\overline{C}} + \overline{\overline{A}+\overline{D}}}} =$$

3 1

$$= \overline{\overline{\overline{\overline{A+A} + \overline{B+B} + \overline{C+C}} + \overline{\overline{A+A} + \overline{D+D}}} + 0}$$

4+6

(1) $\overline{A \,.\, B} = \overline{A} + \overline{B}$

(2) $\overline{A + B} = \overline{A} \,.\, \overline{B}$

(3) $A = \overline{\overline{A}}$

(4) $A + A = A$

(5) $A \,.\, A = A$

(6) $A + 0 = A$

(7) $A \,.\, 1 = A$

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Algebraic forms of functions

Two possible forms:

- SOP (sum of products) or
- POS (product of sums).

(sum = Oring product = ANDing)

Examples:

$f_1(A,B,C) = A.B + B.C$ $f_2(A,B,C)=(A+B).(B+C)$

Canonical forms

Two possible forms:

- Canonical SOP (sum of minterms) or
- Canonical POS (product of maxterms).

(sum = Oring product = ANDing)

minterm = product containing all variables

maxterm = sum containing all variables

Defining minterms

2 variable function

A	B	expression equiv.	
0	0	$\overline{A}.\overline{B}$	m_0
0	1	$\overline{A}.B$	m_1
1	0	$A.\overline{B}$	m_2
1	1	$A.B$	m_3

4 variable function

A B C D	
0 0 0 0	m_0
0 0 0 1	m_1
0 0 1 0	m_2
0 0 1 1	m_3
0 1 0 0	m_4
0 1 0 1	m_5
0 1 1 0	m_6
0 1 1 1	m_7
1 0 0 0	m_8
1 0 0 1	m_9
1 0 1 0	m_{10}
1 0 1 1	m_{11}
1 1 0 0	m_{12}
1 1 0 1	m_{13}
1 1 1 0	m_{14}
1 1 1 1	m_{15}

minterm = product containing all variables
0→complemented variable
1→uncomplemented variable

Defining a function using sum of minterms

A	B	expression equiv.	
0	0	$\overline{A}.\overline{B}$	m_0
0	1	$\overline{A}.B$	m_1
1	0	$A.\overline{B}$	m_2
1	1	$A.B$	m_3

A	B	K
0	0	0
0	1	0
1	0	1
1	1	0

$K = m_2 = \Sigma\, m(2)$

$K = A\,.\,\overline{B}$

2 variable function

Defining maxterms

2 variable function

A	B		expression equiv.
0	0	M_0	$A+B$
0	1	M_1	$A+\overline{B}$
1	0	M_2	$\overline{A}+B$
1	1	M_3	$\overline{A}+\overline{B}$

maxterm = sum containing all variables
0→uncomplemented variable
1→complemented variable

4 variable function

A B C D	
0 0 0 0	M_0
0 0 0 1	M_1
0 0 1 0	M_2
0 0 1 1	M_3
0 1 0 0	M_4
0 1 0 1	M_5
0 1 1 0	M_6
0 1 1 1	M_7
1 0 0 0	M_8
1 0 0 1	M_9
1 0 1 0	M_{10}
1 0 1 1	M_{11}
1 1 0 0	M_{12}
1 1 0 1	M_{13}
1 1 1 0	M_{14}
1 1 1 1	M_{15}

Defining a function using product of maxterms

A	B		expression equiv.
0	0	M_0	$A+B$
0	1	M_1	$A+\overline{B}$
1	0	M_2	$\overline{A}+B$
1	1	M_3	$\overline{A}+\overline{B}$

A	B	K
0	0	0
0	1	0
1	0	1
1	1	0

$K = M_0.M1.M_3=$
$=\Pi\ M(0,1,3)$

$K = (A+B).(A+\overline{B}).(\overline{A}+\overline{B})$

2 variable function

Defining a function using 1's and 0's

A	B	1's equiv.	0's equiv.
0	0	$\overline{A}.\overline{B}$	$A+B$
0	1	$\overline{A}.B$	$A+\overline{B}$
1	0	$A.\overline{B}$	$\overline{A}+B$
1	1	$A.B$	$\overline{A}+\overline{B}$

A	B	K
0	0	0
0	1	0
1	0	1
1	1	0

$K = A\ .\ \overline{B}$

$K = (A+B).(A+\overline{B}).(\overline{A}+\overline{B})$

Defining a function using 1's and 0's

A	B	1's equiv.	0's equiv.
0	0	$\overline{A}.\overline{B}$	$A+B$
0	1	$\overline{A}.B$	$A+\overline{B}$
1	0	$A.\overline{B}$	$\overline{A}+B$
1	1	$A.B$	$\overline{A}+\overline{B}$

A	B	K
0	0	0
0	1	0
1	0	1
1	1	0

$K = A \,.\, \overline{B}$

= ?

$K = (A+B).(A+\overline{B}).(\overline{A}+\overline{B})$

$K = (A+B).(A+\overline{B}).(\overline{A}+\overline{B}) = (A.A+A.\overline{B}+A.B+B.\overline{B}).(\overline{A}+\overline{B}) = (A+A.(\overline{B}+B)+0).(\overline{A}+\overline{B}) =$
$= (A+A.1).(\overline{A}+\overline{B}) = (A+A).(\overline{A}+\overline{B}) = A.(\overline{A}+\overline{B}) = A.\overline{A}+A.\overline{B} = 0+A.\overline{B} = A.\overline{B}$

A	B	$A.\overline{B}$	$A+B$	$A+\overline{B}$	$\overline{A}+\overline{B}$	$(A+B).(A+\overline{B}).(\overline{A}+\overline{B})$
0	0	0	0	1	1	0
0	1	0	1	0	1	0
1	0	1	1	1	1	1
1	1	0	1	1	0	0