36.

i. $\frac{\partial f_1}{\partial x} = -\sin(\log(xy))\frac{1}{xy}y \quad \frac{\partial f_1}{\partial y} = -\sin(\log(xy))\frac{1}{xy}x$

ii. $\frac{\partial f_2}{\partial x} = yx^{y-1} \quad \frac{\partial f_2}{\partial y} = x^y \log(x)$

iii. $\frac{\partial f_3}{\partial x} = \frac{4y^2-4x^2}{(x^2+y^2)^2} \quad \frac{\partial f_3}{\partial y} = \frac{-8xy}{(x^2+y^2)^2}$

iv. $\frac{\partial f_4}{\partial x} = 4y^2e^{-y^2} \quad \frac{\partial f_4}{\partial y} = (8xy - 8xy^3)e^{-y^2}$

v. $\frac{\partial f_5}{\partial x} = 4xy^3z^4 - 26xy \quad \frac{\partial f_5}{\partial y} = 6x^2y^2z^4 - 13x^2 \quad \frac{\partial f_5}{\partial z} = 8x^2y^3z^3$

vi. $\frac{\partial f_6}{\partial x} = 4ze^{-\frac{1}{x^2+y^2+z^2}} + 4xze^{-\frac{1}{x^2+y^2+z^2}}\frac{2x}{(x^2+y^2+z^2)^2} \quad \frac{\partial f_6}{\partial y} = 4xze^{-\frac{1}{x^2+y^2+z^2}}\frac{2y}{(x^2+y^2+z^2)^2}$
$\frac{\partial f_6}{\partial z} = 4xe^{-\frac{1}{x^2+y^2+z^2}} + 4xze^{-\frac{1}{x^2+y^2+z^2}}\frac{2z}{(x^2+y^2+z^2)^2}$

vii. $\frac{\partial f_7}{\partial x} = \cos(\sqrt{w^2+x^2+2y^2+3z^2})\frac{2x}{2\sqrt{w^2+x^2+2y^2+3z^2}}$
$\frac{\partial f_7}{\partial y} = \cos(\sqrt{w^2+x^2+2y^2+3z^2})\frac{4y}{2\sqrt{w^2+x^2+2y^2+3z^2}}$
$\frac{\partial f_7}{\partial z} = \cos(\sqrt{w^2+x^2+2y^2+3z^2})\frac{6z}{2\sqrt{w^2+x^2+2y^2+3z^2}}$
$\frac{\partial f_7}{\partial w} = \cos(\sqrt{w^2+x^2+2y^2+3z^2})\frac{2w}{2\sqrt{w^2+x^2+2y^2+3z^2}}$

viii. $\frac{\partial f_8}{\partial x} = -g(x) \quad \frac{\partial f_8}{\partial y} = g(y)$

37

i. $\frac{\partial^2 f}{\partial z \partial x} = \frac{(x^2+y^2+z^2)2zy-(y^3+z^2y-x^2y)4z}{(x^2+y^2+z^2)^3}$

ii. $\frac{\partial^2 f}{\partial y \partial z} = \frac{-2x^3z+6xy^2z-2xz^3}{(x^2+y^2+z^2)^3}$

iii. $\frac{\partial^2 f}{\partial z \partial y} = \frac{\partial^2 f}{\partial y \partial z}$

iv. $\frac{\partial^2 f}{\partial z^2} = \frac{(x^2+y^2+z^2)(-2xy)+8xyz^2}{(x^2+y^2+z^2)^3}$

v. $\frac{\partial^3 f}{\partial x \partial y \partial z} = \frac{(x^2+y^2+z^2)(-6x^2z+6y^2z-2z^3)-(-2x^3z+6xy^2z-2xz^3)6x}{(x^2+y^2+z^2)^4}$

39 $\frac{\partial f}{\partial x}(0,0) = 1$

40 $\frac{\partial f}{\partial x}(0,0) = 1 \quad \frac{\partial f}{\partial y}(0,0) = -\frac{1}{2}$
A função não é diferenciável em $(0,0)$.

41 $\frac{\partial f}{\partial x}(0,0) = 0 \quad \frac{\partial f}{\partial y}(0,0) = 0$

42
a. $\nabla f(0,0) = [0 \quad 0]^\top$

$\nabla f(x,y) = [\frac{4x^2y^3+yx^4-y^5}{(x^2+y^2)^2} \quad \frac{x^5-4x^3y^2-xy^4}{(x^2+y^2)^2}]^\top$, se $(x,y) \neq (0,0)$

f é continuamente derivável em $\mathbb{R}^2$ logo f é diferenciável em $\mathbb{R}^2$

b. $\frac{\partial^2 f}{\partial x \partial y}(0,0) = 1 \quad \frac{\partial^2 f}{\partial y \partial x}(0,0) = -1$

43
i. $f'_{(3,-2)}(-2,1) = -56 \quad f'_{(\frac{3}{\sqrt{13}},-\frac{2}{\sqrt{13}})}(-2,1) = -\frac{56}{\sqrt{13}}$
ii. $f'_{(-1,2)}(-2,3) = -20 \quad f'_{(-\frac{1}{\sqrt{5}},\frac{2}{\sqrt{5}})}(-2,3) = -\frac{20}{\sqrt{5}}$
iii. $f'_{(1,2,1)}(-2,2,1) = -\frac{1}{9} \quad f'_{(\frac{1}{\sqrt{6}},\frac{2}{\sqrt{6}},\frac{1}{\sqrt{6}})}(-2,2,1) = -\frac{1}{9\sqrt{6}}$

45 $d = [-1 \quad -2.4]^\top$

46 $\frac{\partial f}{\partial x}(P) < 0 \quad \frac{\partial f}{\partial y}(P) > 0$

47
a. f é continua em $(0,1)$
b. f é diferenciável em $(0,1)$
c. $f'_{(2,1)}(0,1) = 1$
d. $f(0.012, 1.005) \approx 1.005$

48 $f'_{(u_1,u_2)}(a,b) = \frac{u_1 u_2^2}{u_1^2}$, se $u_1 \neq 0$

$f'_{(0,1)}(a,b) = 0$

f não é diferenciável em (a,b) e também não é contínua em (a,b)

49 Não é possível prolongar f por diferenciabilidade ao ponto $(0,0)$

$\bar{f}'_{(1,1)}(0,0) = \frac{1}{2}$

50 $f(2+h_1, 1+h_2) \approx 3 - \frac{2}{3}h_1 - \frac{7}{3}h_2$

$f(1.95, 1.08) \approx 2.85$

51 $|E| \leq 0.05$

52 $|E| \leq \frac{44}{25}$

53 $|E| \leq \frac{4\pi}{100}\sqrt{\frac{10}{9.81}}$

56 b. O maior aberto onde as derivadas parciais de primeira ordem de f estão definidas é $D = \{(x,y) \in \mathbb{R}^2 : x \neq 0\}$

60

a. $\frac{\partial f}{\partial x} = -\frac{\partial F}{\partial u} y \sin(x) + \frac{\partial F}{\partial v} e^{-x^2y^2} y$

$\frac{\partial f}{\partial y} = \frac{\partial F}{\partial u} \cos(x) + \frac{\partial F}{\partial v} e^{-x^2y^2} x$

b. $\frac{\partial g}{\partial x} = \frac{\partial F}{\partial G}\frac{\partial G}{\partial x} + \frac{\partial F}{\partial v}\frac{y^2}{y^4+x^2}$

$\frac{\partial g}{\partial y} = \frac{\partial F}{\partial G}\frac{\partial G}{\partial y} + \frac{\partial F}{\partial v}\frac{-2xy}{y^4+x^2}$

62

a. $Jac\, g(0,0,0) = \begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$

$g(0.01, 0.2, 0.03) \approx [1.03 \quad 0.08]^\top$

b. $Jac\,(f \circ g)(1,1,1) = \begin{bmatrix} \frac{\pi}{2} + 4 + \frac{(1+e)^2}{6} & \frac{(1+e)^2}{6} & (\frac{\pi}{4}+2)e + \frac{(1+e)^2}{6} \\ \sqrt{2} + \frac{e^{1+e}}{6} & \frac{e^{1+e}}{6} & \frac{\sqrt{2}}{2}e + \frac{e^{1+e}}{6} \end{bmatrix}$

63

$$Jac\, f(x_1,x_2,x_3,y_1,y_2,y_3) = \begin{bmatrix} y_1 & y_2 & y_3 & x_1 & x_2 & x_3 \end{bmatrix}$$

$$Jac\, g(x_1,x_2,x_3,y_1,y_2,y_3) = \begin{bmatrix} 0 & y_3 & -y_2 & 0 & -x_3 & x_2 \\ -y_3 & 0 & y_1 & x_3 & 0 & -x_1 \\ y_2 & -y_1 & 0 & -x_2 & x_1 & 0 \end{bmatrix}$$

Para qualquer uma das funções, as derivadas parciais são contínuas logo as funções são continuamente deriváveis e como tal diferenciáveis.