TÓPICOS DA RESOLUÇÃO DO 1º TESTE DE AM2E (25/10/2014)

1.

a) $xy' - 2y = x^3 \cos x \Leftrightarrow y' - \frac{2}{x} y = x^2 \cos x$

25/10

$$G(x) = \int \left(-\frac{2}{x}\right) dx = -2 \log x = \log x^{-2}$$

Factor integrante é $\mu(x) = e^{\log x^{-2}} = x^{-2}$

Temos

$$y' - \frac{2}{x} y = x^2 \cos x \Leftrightarrow x^{-2} y' - 2x^{-3} y = \cos x$$

$$\Leftrightarrow (x^{-2} y)' = \cos x \Leftrightarrow x^{-2} y = \int \cos x \, dx \Leftrightarrow$$

$$\Leftrightarrow \quad x^{-2}y = \operatorname{sen} x + c \Leftrightarrow y = x^2 \operatorname{sen} x + cx^2, \ c \in \mathbb{R}$$

Dado que $y(\pi/2) = 1$ vem $1 = (\pi/2)^2 \operatorname{sen} \pi/2 + c(\pi/2)^2$

ou seja $1 = \frac{\pi^2}{4} + c\frac{\pi^2}{4}$. Logo $c = \frac{4-\pi^2}{\pi^2}$.

Assim a solução é $y = x^2 \operatorname{sen} x + \frac{4-\pi^2}{\pi^2} x^2$.

b) $u = 2x + 2y + 1 \Leftrightarrow y = \frac{u - 2x - 1}{2}$. Donde fazendo a mudança de variável vem

$$\left(\frac{u-2x-1}{2}\right)' = u^2 \Leftrightarrow \frac{1}{2}(u'-2) = u^2 \Leftrightarrow$$

$$u' - 2 = 2u^2 \iff u' = 2u^2 + 2 \iff \frac{du}{2u^2+2} = dx$$

$$\iff \frac{1}{2} \int \frac{1}{u^2+1}\, du = \int 1\, dx \iff$$

$$\iff \frac{1}{2} \operatorname{arctg} u = x + C$$

Então temos

$$\frac{1}{2} \operatorname{arctg} (2x+2y+1) = x + C \quad \text{(integral geral)}$$

2. a) $y' + (x+2)y^2 = 0 \Leftrightarrow y' = -(x+2)y^2$

Método de Euler $\Delta x = 0{,}2$ $y\left(\frac{4}{10}\right)$ aproximado

n	x_n	y_n	$f(x_n, y_n)\,\Delta x$	y_{n+1}
0	0	1	-0.4	0,6
1	0,2	0,6	-0.1584	0,4416

Assim temos $y(0.4) \approx 0{,}4416 = y_2$.

b) Dado que $y' = -(x+2)y^2$ temos

$$y'(1) = -(1+2)\left(\frac{2}{7}\right)^2 = -\frac{12}{49}.$$

Logo a recta tangente ao gráfico de g no ponto $(1, 2/7)$ é

$$y - 2/7 = -\frac{12}{49}(x-1)$$

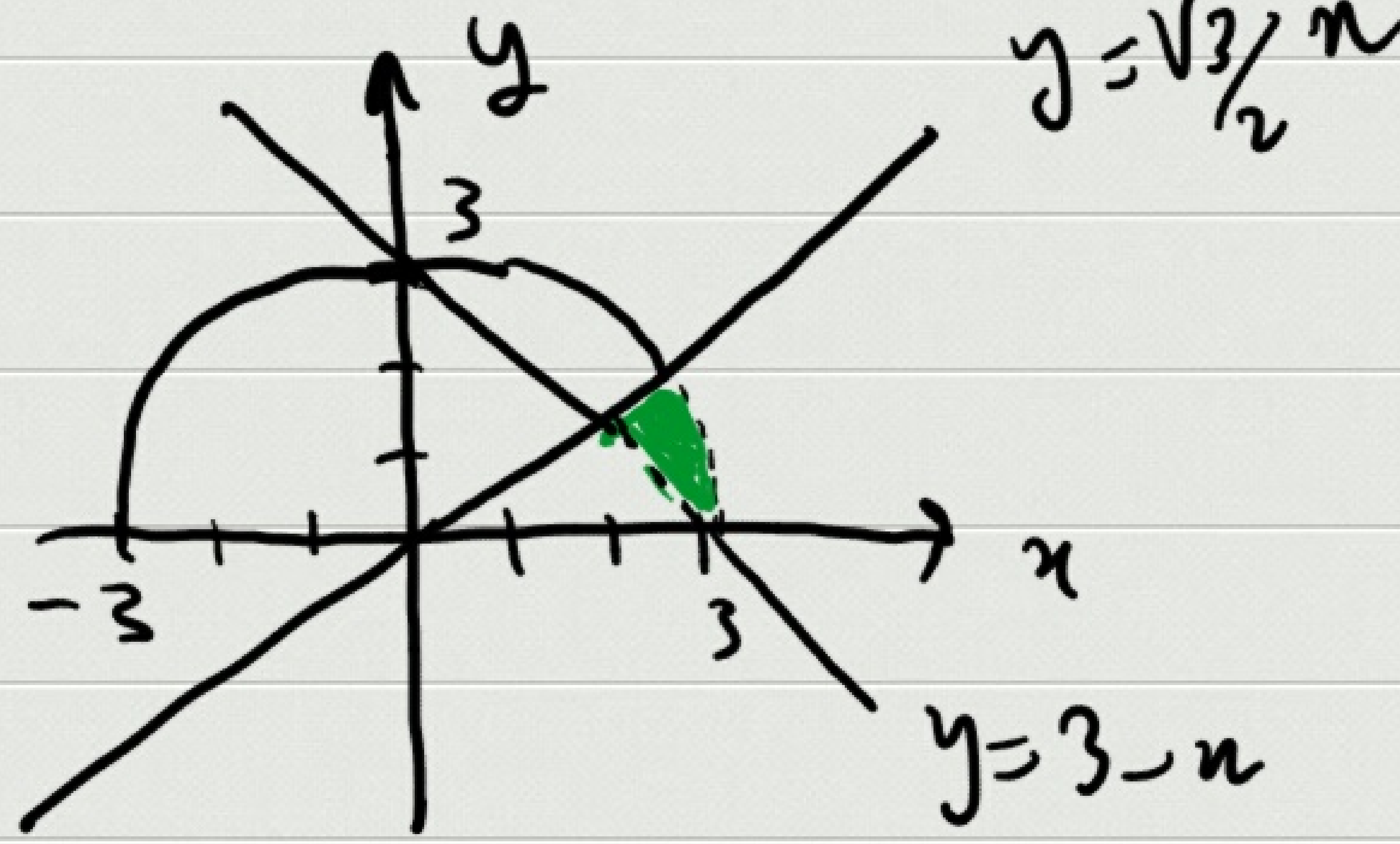
3.

a) $$y < \sqrt{9-x^2}, \quad 3-x < y \leq \frac{\sqrt{3}}{2}x$$

$$y = \sqrt{9-x^2} \Leftrightarrow y^2 = 9-x^2 \wedge y \geq 0 \Leftrightarrow x^2+y^2 = 9 \wedge y \geq 0$$

$$y = \frac{\sqrt{3}}{2}x \quad \text{e} \quad \frac{\sqrt{3}}{3} < \frac{\sqrt{3}}{2} < 1$$

donde $\pi/6 < \operatorname{arctg}\left(\frac{\sqrt{3}}{2}\right) < \pi/4$.



$y=3-x$

$y=\frac{\sqrt{3}}{2}x$

$x^2+y^2=9 \wedge y\geq 0$

b) $y>3-x \Leftrightarrow r\,\text{sen}\theta > 3-r\cos\theta \qquad \theta\in[0,\pi/2]$

$\Leftrightarrow r\,\text{sen}\theta + r\cos\theta > 3$

$\Leftrightarrow r(\text{sen}\theta+\cos\theta)>3$

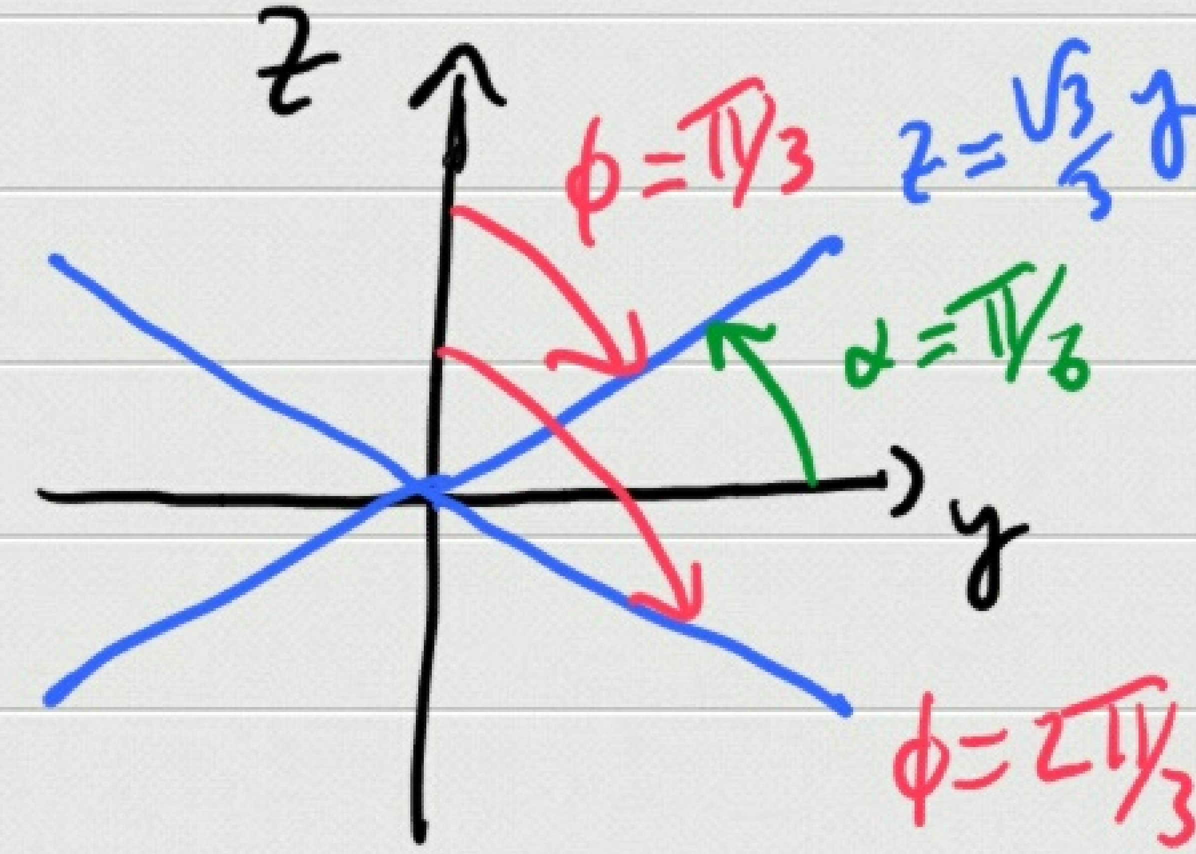
$\Leftrightarrow r > \dfrac{3}{\text{sen}\theta+\cos\theta} \qquad (\text{sen}\theta+\cos\theta>0)$

$$y \leq \frac{\sqrt{3}}{2} x \iff 0 \leq \theta \leq \operatorname{arctg}\left(\frac{\sqrt{3}}{2}\right)$$

$$\frac{3}{\cos\theta + \operatorname{sen}\theta} < r \leq 3$$

4.

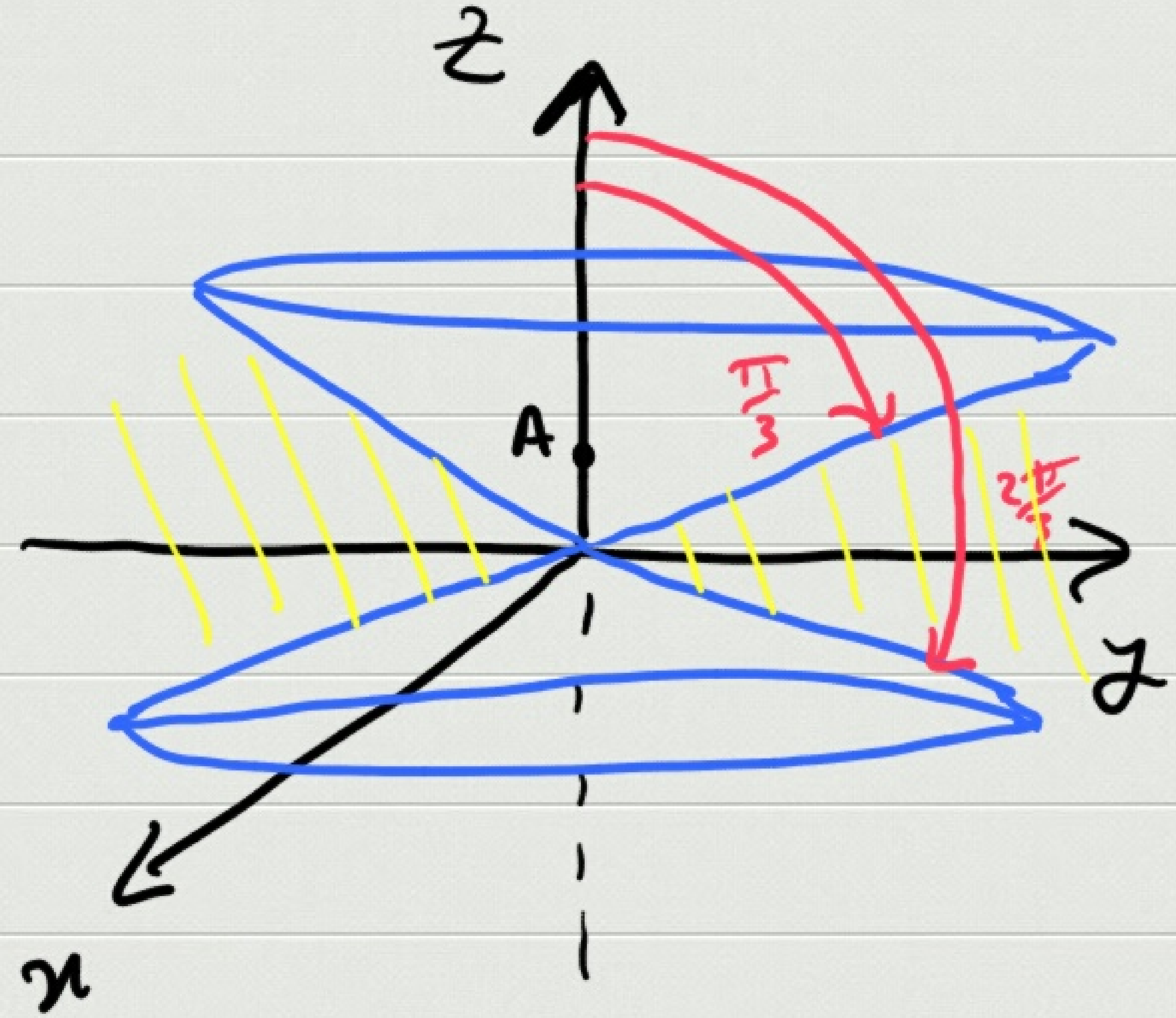
a) $z^2 = \frac{1}{3}(x^2+y^2) \iff z = \pm \frac{\sqrt{3}}{3}\sqrt{x^2+y^2}$ superfície cónica dupla



Secção $x = 0$

$z = \frac{\sqrt{3}}{3} y$ ou $z = -\frac{\sqrt{3}}{3} y$

$m = \frac{\sqrt{3}}{3} \Rightarrow \operatorname{tg}\alpha = \frac{\sqrt{3}}{3} \Rightarrow \alpha = \pi/6$



$$\frac{\pi}{3} \leq \phi \leq 2\frac{\pi}{3}$$

$\boxed{z^2 \leq \frac{1}{3}(x^2+y^2)}$ → região sólida no exterior das duas superfícies cónicas

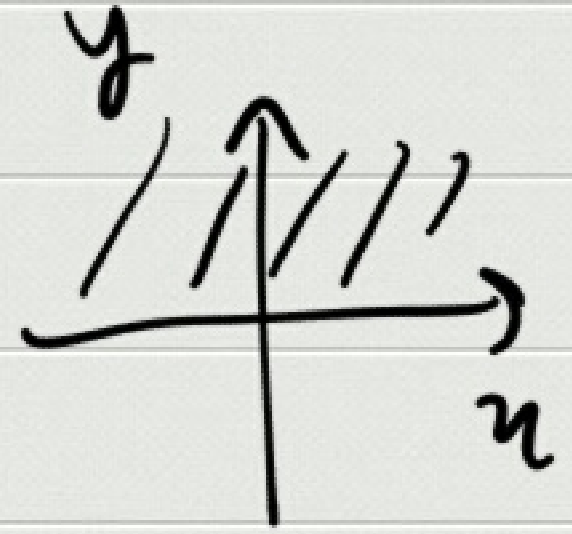
"$A=(0,0,1) \notin z^2 \leq \frac{1}{3}(x^2+y^2)$" pois $1 \not\leq 0$.

• $z^2 < \frac{1}{3}(x^2+y^2)$ $\rightarrow$ região sólida pro exterior dos cones.

[Em alternativa: $z^2 < \frac{1}{3}(x^2+y^2) \Leftrightarrow \rho^2 \cos^2\phi < \frac{1}{3}\rho^2 \sin^2\phi$

$\Leftrightarrow \operatorname{tg}^2\phi > 3 \Leftrightarrow \frac{\pi}{3} < \phi < \frac{2\pi}{3}$.]

• $y \geqslant 0$



$\theta \in [0, \pi]$

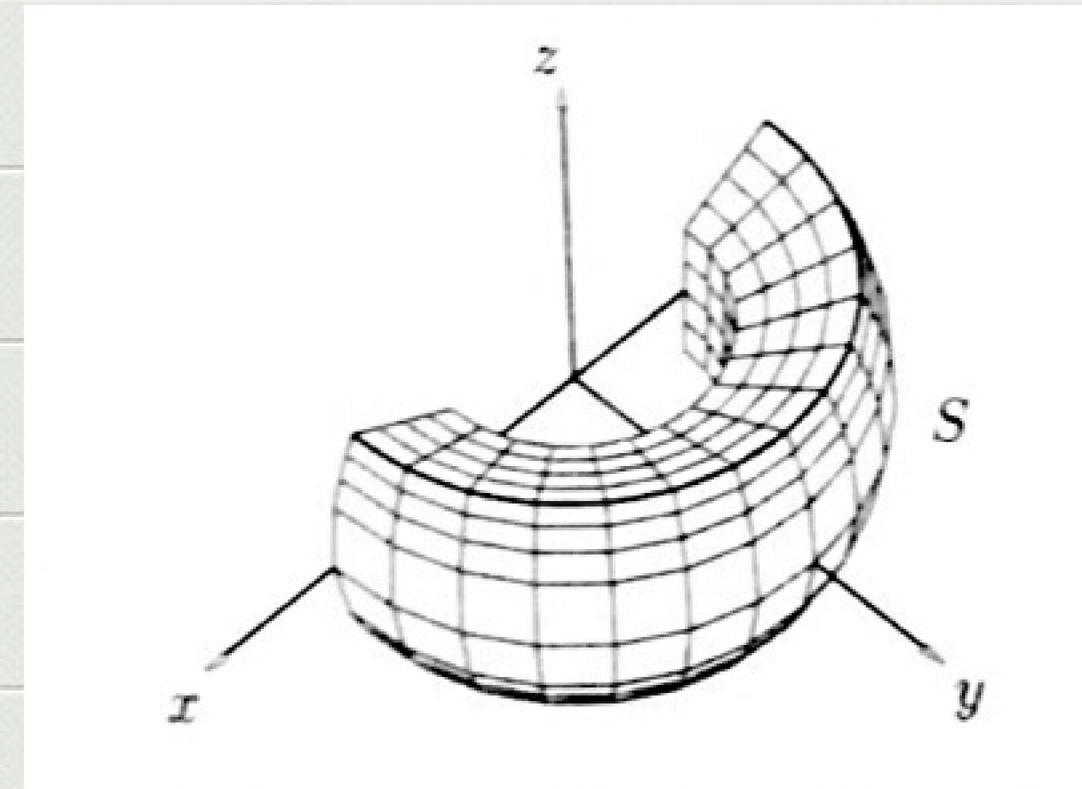
• $x^2+y^2 > 1 \Leftrightarrow \rho^2 \sin^2\theta \cos^2\phi + \rho^2 \sin^2\theta \sin^2\phi > 1 \Leftrightarrow$

$\Leftrightarrow \rho^2 \sin^2\theta > 1 \Leftrightarrow \rho > \frac{1}{\sin\theta}$

• $x^2+y^2+z^2 < 4 \Leftrightarrow \rho \leq 2$

Então

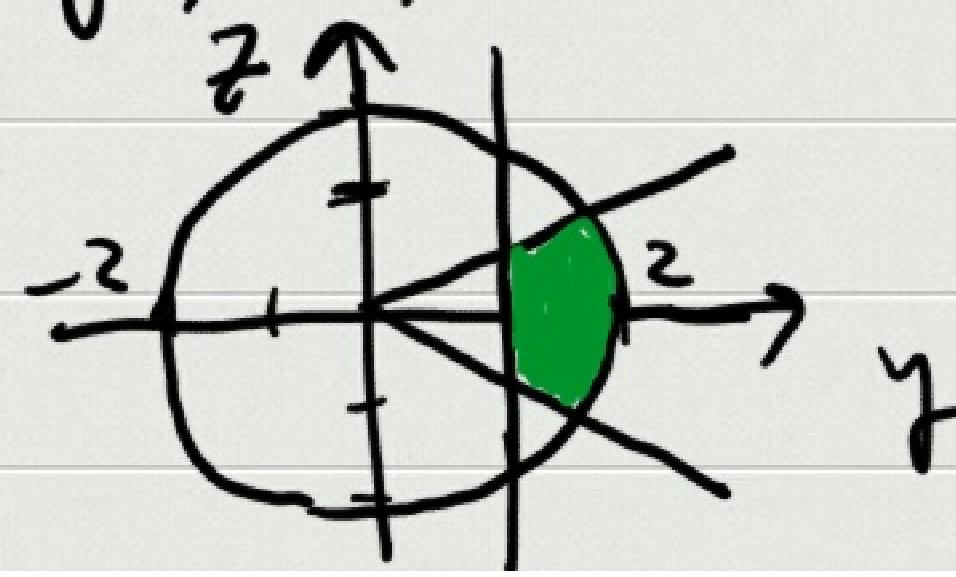
$$S = \left\{ (\rho, \theta, \phi) : \frac{1}{\sin\theta} < \rho < 2 \, , \, 0 \leq \theta \leq \pi \, , \, \frac{\pi}{3} < \phi < \frac{2\pi}{3} \right\}$$



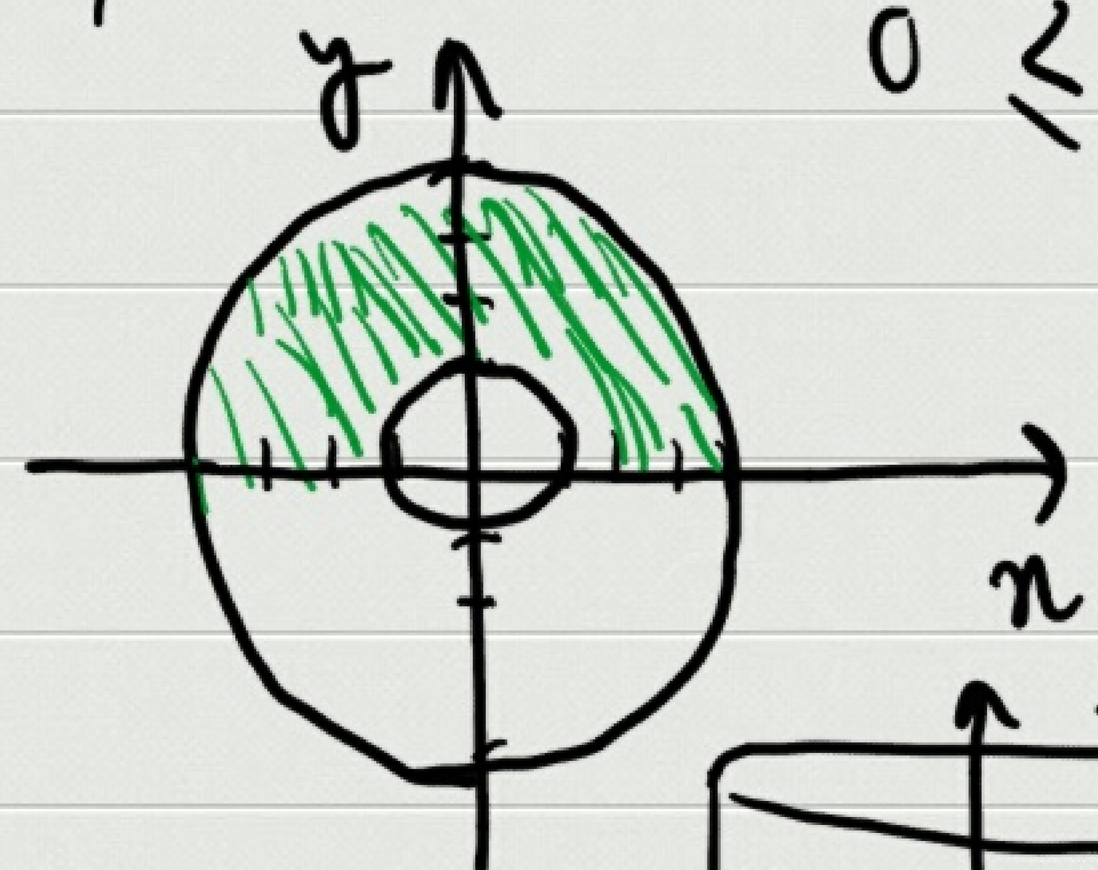
b) <u>secção $x=0$</u>

$z^2 \leq \frac{1}{3} y^2$, $y^2 + z^2 \leq 4$, $y^2 \geq 1$ e $y \geq 0$, onde fica

$z \leq \frac{1}{3} |y|$, $y^2 + z^2 \leq 4$, $y \geq 1$.



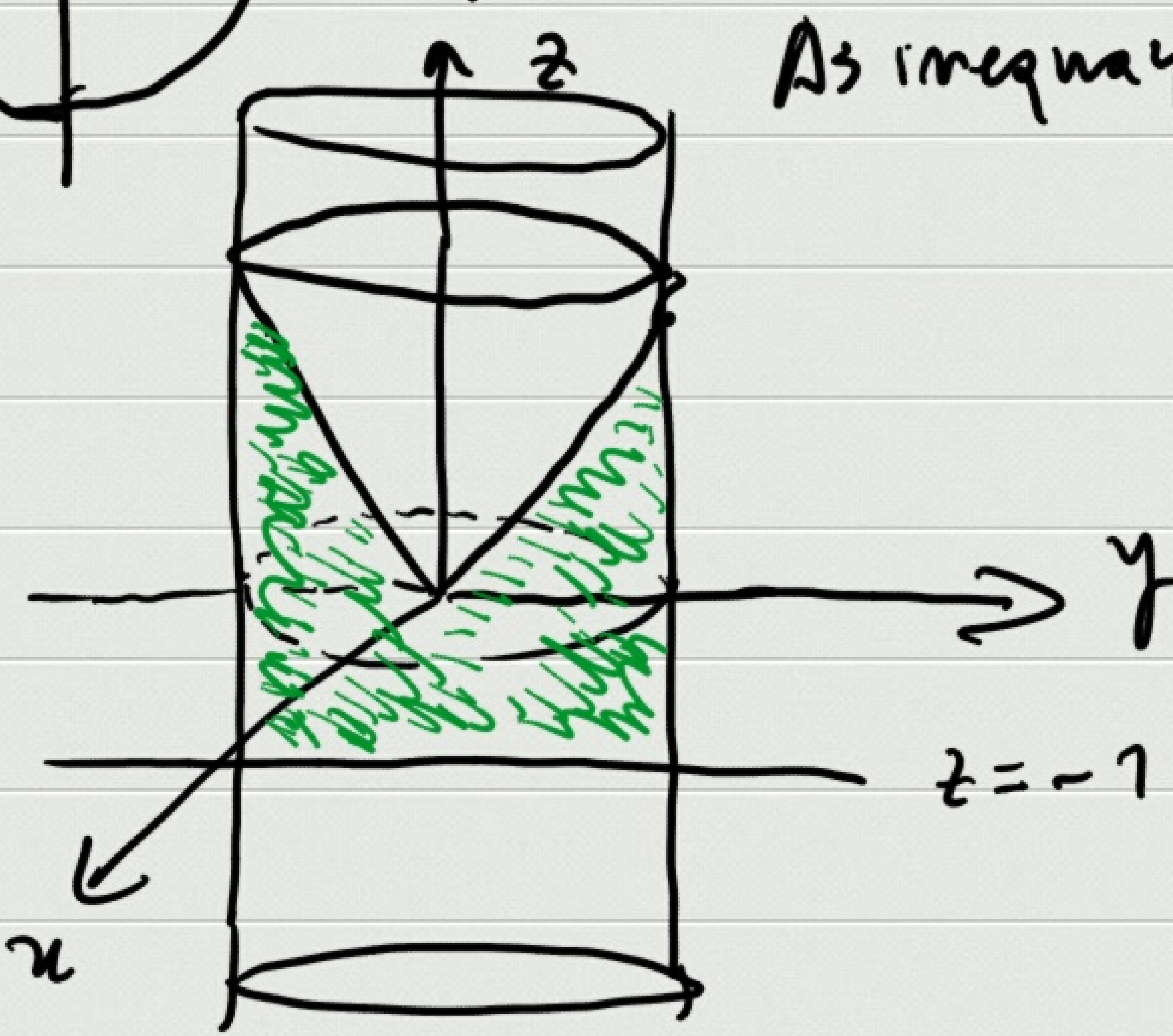
Secção $z=0$



$0 \le \frac{1}{3}(x^2+y^2)$, $x^2+y^2 \le 4$, $x^2+y^2 \ge 1$ e $y \ge 0$.

ou seja $1 \le x^2+y^2 \le 4$ e $y \ge 0$.

e)



As inequações são

$x^2+y^2 \le 1 \rightarrow$ dentro do cilindro $x^2+y^2=1$.

$z \ge -1 \rightarrow$ acima do plano $z=-1$.

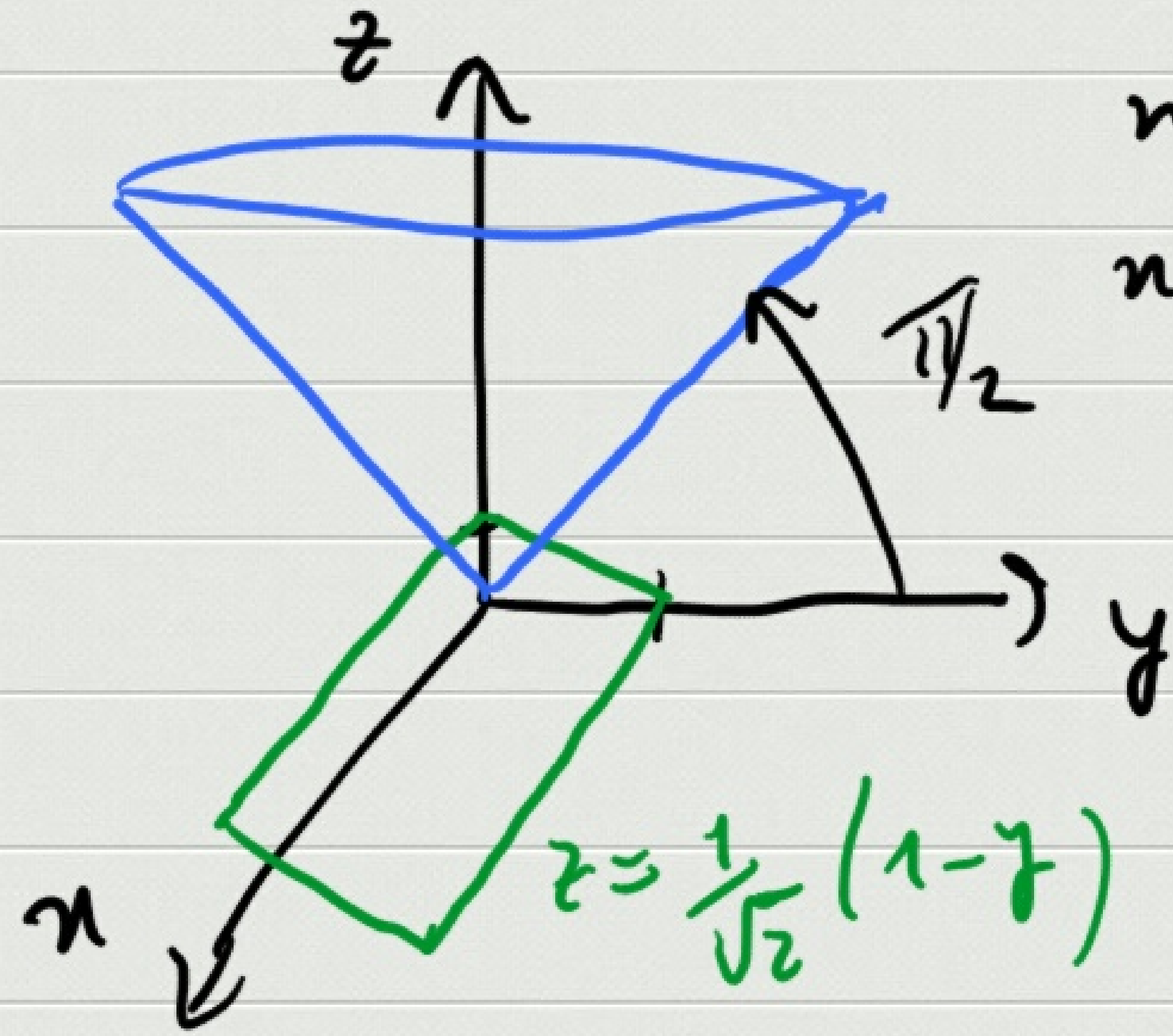
$z \le \frac{\sqrt{3}}{3}\sqrt{x^2+y^2}$

$\rightarrow$ fora do cone $z = \frac{\sqrt{3}}{3}\sqrt{x^2+y^2}$

5.

a) $z = \sqrt{x^2+y^2}$ superfície cónica

$z = \frac{1}{\sqrt{2}}(1-y)$ plano



$x = y = 0 \Rightarrow z = \sqrt{2}/2$

$x = z = 0 \Rightarrow y = 1$

b)

i) $\begin{cases} z = \sqrt{x^2+y^2} \\ z = \frac{1}{\sqrt{2}}(1-y) \end{cases} \Leftrightarrow \begin{cases} \underline{\sqrt{x^2+y^2} = \frac{1}{\sqrt{2}}(1-y)} \\ \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} \underline{x^2+y^2 = \frac{1}{2}}(1-y)^2 \\ \end{cases} \Leftrightarrow \begin{cases} x^2+y^2 = \frac{1}{2}(1-2y+y^2) \\ z = \frac{1}{\sqrt{2}}(1-y) \end{cases}$

$2x^2+2y^2 = 1-2y+y^2 \Leftrightarrow 2x^2+y^2+2y = 1$

$\Leftrightarrow 2x^2+(y+1)^2 = 2 \Leftrightarrow x^2+\frac{(y+1)^2}{2} = 1$

$\Leftrightarrow x^2+\left(\frac{y+1}{\sqrt{2}}\right)^2 = 1$

$$\begin{cases} x = \cos t \\ \dfrac{y+1}{\sqrt{2}} = \operatorname{sen} t \\ z = \dfrac{1}{\sqrt{2}}(1-y) \end{cases} \Leftrightarrow \begin{cases} x = \cos t \\ y = \sqrt{2}\operatorname{sen} t - 1 \\ z = \sqrt{2} - \operatorname{sen} t \end{cases}$$

$$\sigma(t) = (\cos t, \sqrt{2}\operatorname{sen} t - 1, \sqrt{2} - \operatorname{sen} t)$$

$$t = 0 \to (x, y, z) = (0, \sqrt{2}-1, \sqrt{2}-1)$$

$$s \xrightarrow{\varphi} s + \pi/2 \xrightarrow{\sigma} (\cos(t+\pi/2), \sqrt{2}\operatorname{sen}(t+\pi/2) - 1, \sqrt{2} - \operatorname{sen}(t+\pi/2))$$

$$r(s) = \sigma \circ \varphi(s), \quad r(0) = (0, \sqrt{2}-1, \sqrt{2}-1).$$

$(x, y, z) = (-1, -1, \sqrt{2}) \Rightarrow \cos(t + \pi/2) = -1$

$\Rightarrow t = \pi/2 + 2k\pi$

Seja $t = \pi/2$

$v(t) = r'(t) = (-\operatorname{sen}(t + \pi/2), \sqrt{2}\cos(t + \pi/2), -\cos t)$

donde $v(\pi/2) = (-\operatorname{sen}\pi, \sqrt{2}\cos\pi, -\cos\pi) =$

$= (0, \sqrt{2}, -1)$

e $\|v(t)\| = \sqrt{2+1} = \sqrt{3}$.

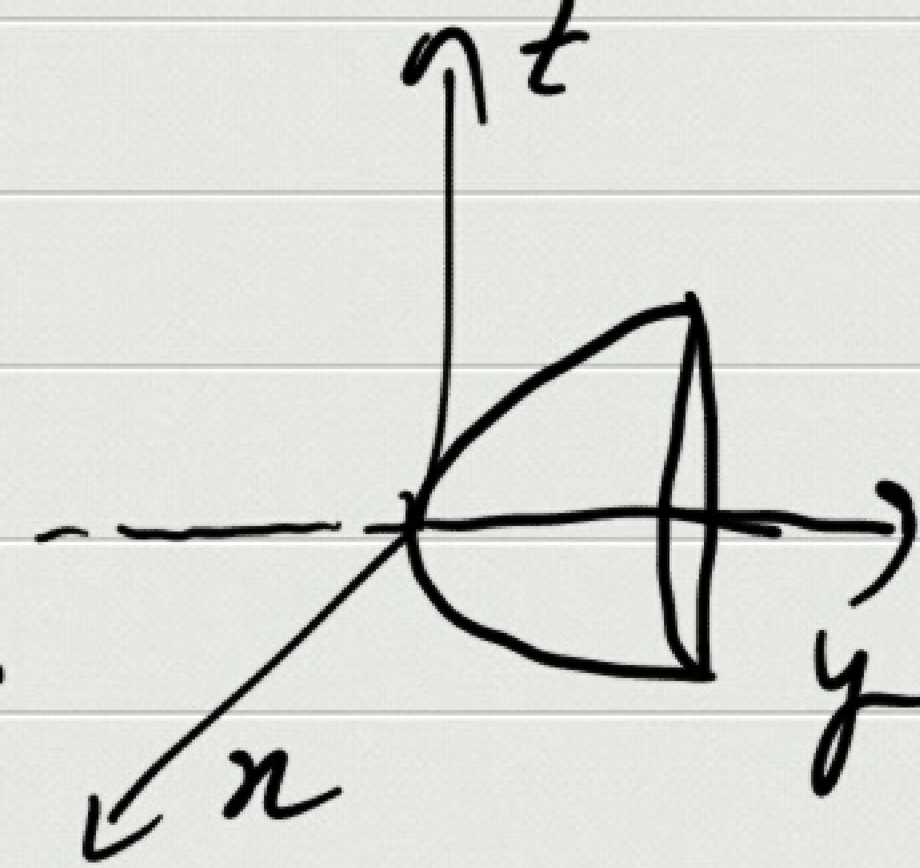
6. a)

$$f(x,y,z) = 2$$

$$\frac{y}{x^2+z^2} = 2 \Leftrightarrow 2(x^2+z^2) = y$$

$$\Leftrightarrow y = \frac{x^2}{\left(\frac{\sqrt{2}}{2}\right)^2} + \frac{z^2}{\left(\frac{\sqrt{2}}{2}\right)^2}$$

É um parabolóide circular em torno do eixo yy.



b) $K=1$ $\quad \frac{x^2}{\left(\frac{\sqrt{2}}{2}\right)^2} + \frac{z^2}{\left(\frac{\sqrt{2}}{2}\right)^2} = 1 \Leftrightarrow x^2+z^2 = \left(\frac{\sqrt{2}}{2}\right)^2$

$K=4$ $$\frac{x^2}{\left(\frac{\sqrt{2}}{2}\right)^2}+\frac{z^2}{\left(\frac{\sqrt{2}}{2}\right)^2}=4 \Leftrightarrow x^2+z^2=8$$

$K=8$ $$\frac{x^2}{\left(\frac{\sqrt{2}}{2}\right)^2}+\frac{z^2}{\left(\frac{\sqrt{2}}{2}\right)^2}=8 \Leftrightarrow x^2+z^2=16$$