Teoria da Computação

MIEI 2018/2019 - FCT UNL

Aula Prática 1

Inductive sets

Provide an inductive definition for the following sets:

1. $PowersOfTwo \stackrel{\text{def}}{=} \{n \in NAT \mid n = 2^p \wedge p \in NAT\}$.

2. $exp2 \in NAT \to NAT$ such that $exp2(n) = 2^n$.

   Give a justification, using derivation trees, that $4 \mapsto 16 \in exp2$ (or equivalently, that $exp2(4) = 16$).

3. $OddSequences \stackrel{\text{def}}{=} \{s \in SEQ \mid len(s)\%2 = 1\}$.

   Give a justification, using derivation trees, that $(1, 2, 3) \in OddSequences$.

4. $SortedSequences$, the set of increasing sequences of natural numbers.

   For example,
   $$(2, 5, 6, 6, 7) \in SortedSequences$$
   $$(1, 2, 1, 3, 4) \notin SortedSequences$$

   Give a justification, using derivation trees, that $(3, 5, 5, 8) \in SortedSequences$.

5. The relation
   $$Reverse \subset SEQ \times SEQ$$
   such that $(s_1, s_2) \in Reverse$ if and only if s_2 is the reverse sequence of s_1. For example,
   $$((1, 3, 5), (5, 3, 1)) \in Reverse$$
   $$((1, 3, 5), (2, 3)) \notin Reverse$$

   Give a justification, using derivation trees, that $((1, 3, 5), (5, 3, 1)) \in Reverse$.