Theoretical Computer Science

DI FCT UNL

1 Jul 2014

1. Define the following sets, using the facilities of basic set theory:

   (a) The set of pairs of natural numbers such that the value of the second component is not larger that the value of the first component.

   (b) The health care card registers the following data: the user name, the user number, and the list of medical events. A medical event contains the following data: the date, the description of the medical event. **Cannot be more than one event each day** in a card. Model the set EV of medical events and the set MED of health care cards.

   (c) Define a function $d : MED \times DATE \to \wp(EV)$ that given a set of health care cards and a date, gives a set of all medical events in that date.

2. Model the tic-tac-toe game with a structure. The board is a 3×3 matrix. The set of board positions is $P = \{1, 2, 3\} \times \{1, 2, 3\}$. Each board cell can be either empty, or contain a symbol in $M = \{\texttt{X}, \texttt{O}\}$. Initially each board cell is empty. A player move writes either `O` or `X` in an empty cell.

   (a) Model the set T of all game states (do not care about legal move sequences).

   (b) Define the function $play : T \times P \times M \to T$ that models then next player move. The function is partial, only defined if the position is empty.

   (c) Define the function $winD : T \to bool$ that returns true if and only if some player won the game with a diagonal.

3. Consider the alphabet (with 5 symbols) $\Sigma = \{\texttt{login}, \texttt{logout}, \texttt{getf}, \texttt{putf}, \texttt{delf}\}$ supposed to model the actions of an user with a dropbox folder. Consider the regular expression

$$(\texttt{login}\ \texttt{getf}^*\ \texttt{logout}) + (\texttt{login}\ (\texttt{putf} + \texttt{delf})^*\ \texttt{logout})$$

Carefully show how the general method for constructing a NFA accepting the same language as a regular expression can be used to systematically derive a NFA corresponding the regular expression above.

4. Carefully show how the general method for constructing a DFA accepting the same language as a NFA can be used to systematically derive a DFA corresponding to the regular expression in 3.

5. Consider the 4 symbol alphabet $\Sigma = \{\texttt{empty}, \texttt{push}, \texttt{pop}, \texttt{notempty}\}$ and the CFG grammer $G = \langle V, \Sigma, S, R\rangle$ where $V = \{S, E, N\}$ and R contains the grammar rules:

$$\begin{array}{lcl} S & \rightarrow & E \\ E & \rightarrow & \texttt{empty}\ E \\ E & \rightarrow & \epsilon \\ E & \rightarrow & \texttt{push}\ N\ \texttt{pop}\ E \\ N & \rightarrow & \texttt{push}\ N\ \texttt{pop}\ N \\ N & \rightarrow & \texttt{notempty}\ N \\ N & \rightarrow & \epsilon \end{array}$$

   (a) Show (if such a thing exists) a derivation in G for the word

   $$\texttt{empty push notempty pop empty}$$

   (b) Check if the grammar shown is LL(1). To that end, construct the transition table for the deterministic syntactic analizer. In the negative case, show in what situation there is a parsing conflict.

6. Remember that a regular language is any language accepted by a DFA. Explain how for any DFA M you can define a CFG (context free grammar) G_M generating the same language that M accepts (Hint: represent M states by G non-terminal symbols and M transitions by G grammar rules).