Light and Illumination

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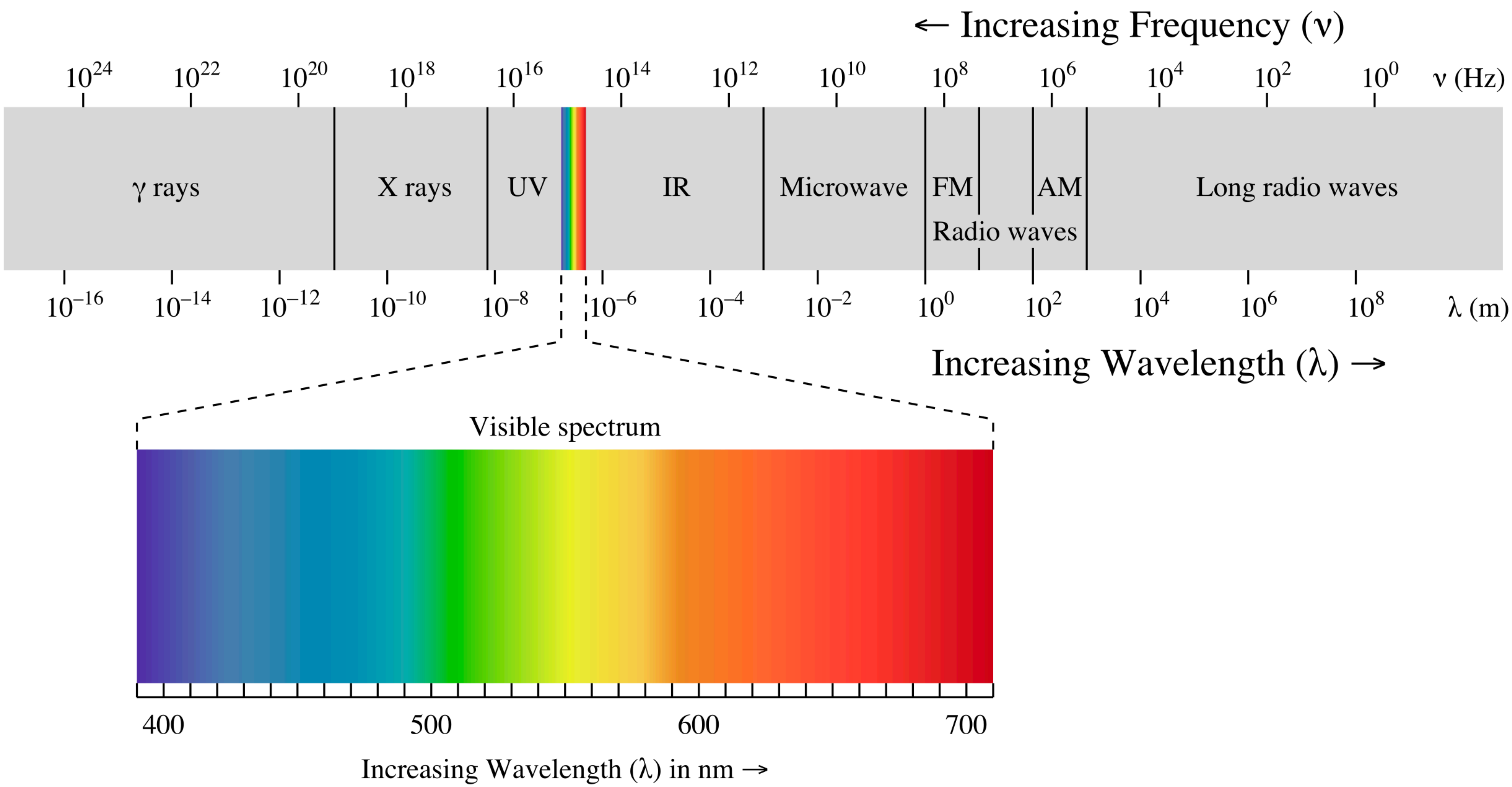
Light

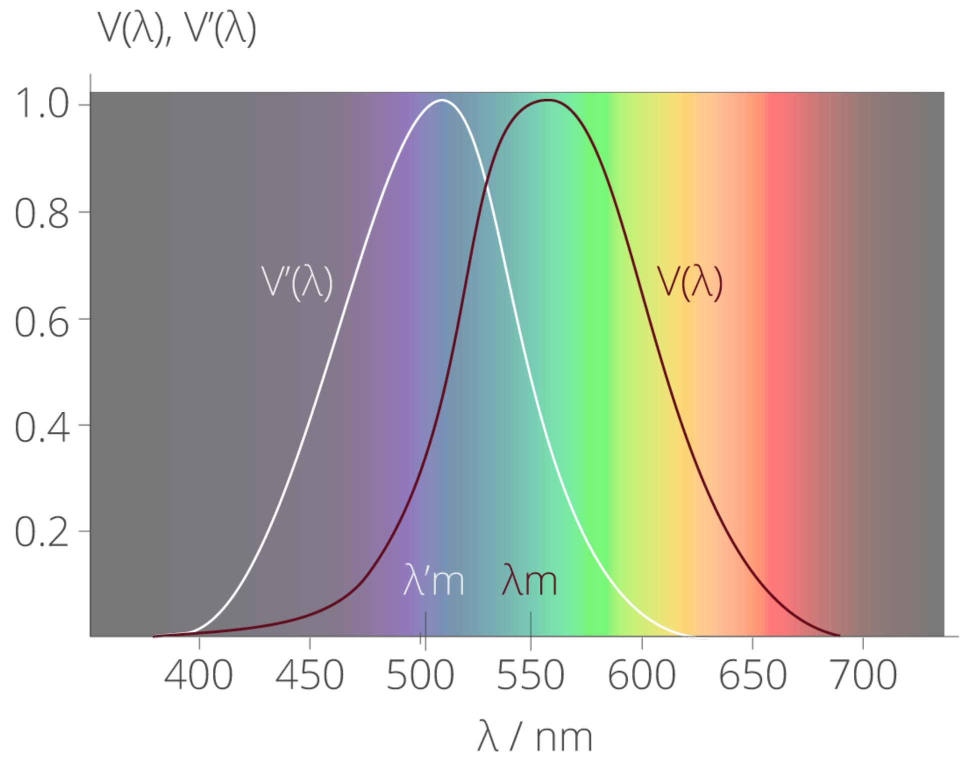
Light



* https://en.wikipedia.org/wiki/Light#/media/File:EM_spectrum.svg

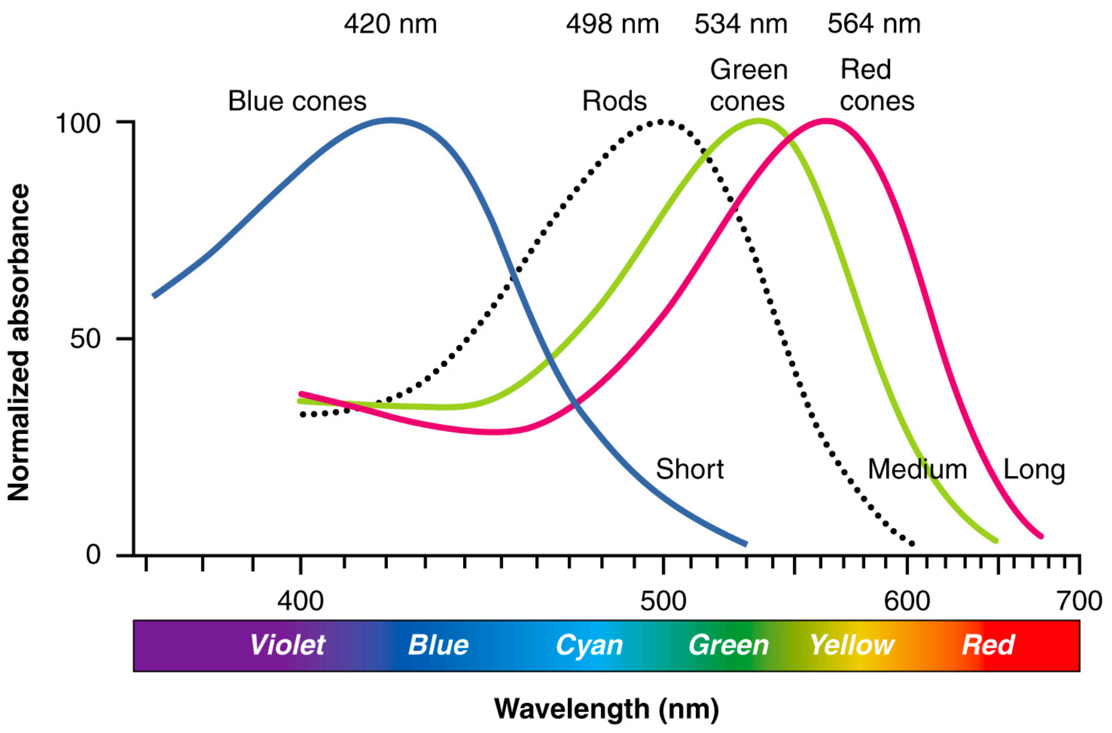
Light Perception

- Sensitivity varies over the spectrum
- In daylight conditions: peak at 555nm (photopic)
- Near darkness: peak at 505nm (scotopic)



Light Perception

- Cones are responsible for our color perception (photopic vision)
- Rods (only one type) are highly sensitive to light (night vision)
- Cones are specialised (3 types)



Models of Light

- Light interaction with materials exhibit different behaviours not easily simulated by a single model.
- Diffraction and interference can be explained by assuming that light is a wave.
- Photoelectric behaviour is better explained recurring to a model consisting of a stream of particles.
- Different computational models are needed to simulate different effects.

Models of Light
Geometric Optics

- Simplest and most commonly used model of light in Compute Graphics.
- wavelength is assumed to be much smaller than the scale of the objects.
- Light is assumed to be emitted, reflected and transmitted.
- Assumptions of the model:
    - Light travels in straight lines (no bending around objects - diffraction)
    - Light travels instantaneously through a medium (infinite speed)
    - No external factors such as gravity (see Einstein and Eddington) or magnetic fields (see Faraday's effect) influence light
- Additional assumptions for simplification: no participating media (fog), no varying indices of refraction (mirage light effects)

Illumination Algorithms

- The purpose of an **Illumination algorithm** is to account for the interaction of light, contributed to a scene by light sources and the objects in that scene
- The light/object interaction relies on an **illumination model** that ultimately defines the colour (intensity) of light that should be seen at a specific point of the scene
- Additionally, a **shading method** is used to determine the colour of every pixel on a surface by using the results of the **illumination model** evaluated at a set of points:
  - interpolated colours: evaluated at a sparser set of points (typically mesh vertices in **per vertex illumination**) and colours are interpolated inside triangles
  - **per pixel illumination**: every pixel belonging to the projection of a surface is evaluated once
- When the illumination model, evaluated at a specific surface point takes into account:
  - only the light arriving directly from each light source, the algorithm is said to be a **local**/direct **illumination algorithm**
  - the contribution from direct sources of light as well as light arriving from other surfaces in the scene (either by reflection/scattering or transmission), the algorithm is said to be a **global**/indirect **illumination algorithm**.

Global Illumination

- The goal of a global illumination algorithm is to compute the steady-state distribution of light in a scene
- **Radiometry**: area of study involved in the physical measurement of light
- **Photometry**: area of study that deals with the quantification of the perception of light energy.
- Human visual system is sensitive to light in the wavelength range of 380nm to 780nm. The sensitivity of the human eye across the visible spectrum has been standardised.
- Photometric terms take this standardisation into account.
- Photometric quantities can be derived from radiometric ones thus, global illumination algorithms operate on radiometric terms rather than photometric ones.
- Photometry is important at the final stage of presenting radiometric information to a user in a display device.

Radiometry

Radiant Power or Flux*

- The fundamental radiometric quantity is radiant power or flux.
- Describes how much total energy flows from/to/through a surface per unit time.

$$\Phi = \frac{dQ}{dt}, [W][J/s]$$

- Examples:
  - A light bulb emits 50 Watt radiant power
  - 20 Watt radiant power is incident on a table
- No information regarding spectral distribution or directional distribution.

* Fluxo Radiante

Irradiance*

- Irradiance (E) is the incident radiant power on a surface, per unit surface area.

$$E = \frac{d\Phi}{dA}, [Wm^{-2}][Js^{-1}m^{-2}]$$

- If 50 Watt radiant power is incident on a 1.25 m^2 surface area, the mean irradiance at each surface point will be 40 $W.m^{-2}$.

* Densidade de fluxo radiante - Irradiância (receptor)

Radiant Exitance or Radiosity*

- Radiant Exitance (M) also called Radiosity (B) is the exitant radiant power per unit surface area.

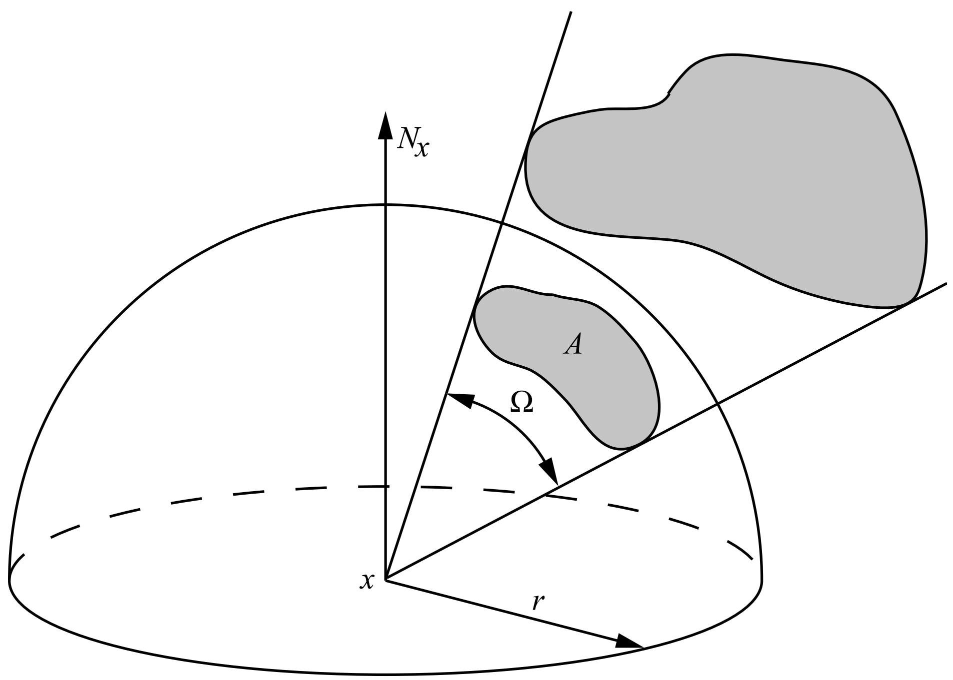
$$M = B = \frac{d\Phi}{dA}, [Wm^{-2}][Js^{-1}m^{-2}]$$

- A light source, of area 0,1m^2, emitting 100 Watt radiant power will have a radiant exitance (radiosity) at each point of its surface of 1000 W.m^{-2}.

* Densidade de fluxo radiante - Emitância (receptor)

Solid angle

- A solid angle **Ω** is the two-dimensional angle in three-dimensional space that an object subtends at a point.
- It is a measure of how large the object appears to an observer looking from that point.
- It is equal to the area of the object projected onto a unit sphere centred at the point.
- Since the area of a sphere is $4\pi r^2$, a solid angle that covers the whole sphere is 4π, and a complete hemisphere is 2π.
- Solid angles are dimensionless but are expressed as steradians (esterradiano ou esferorradiano [PT]).



Radiance

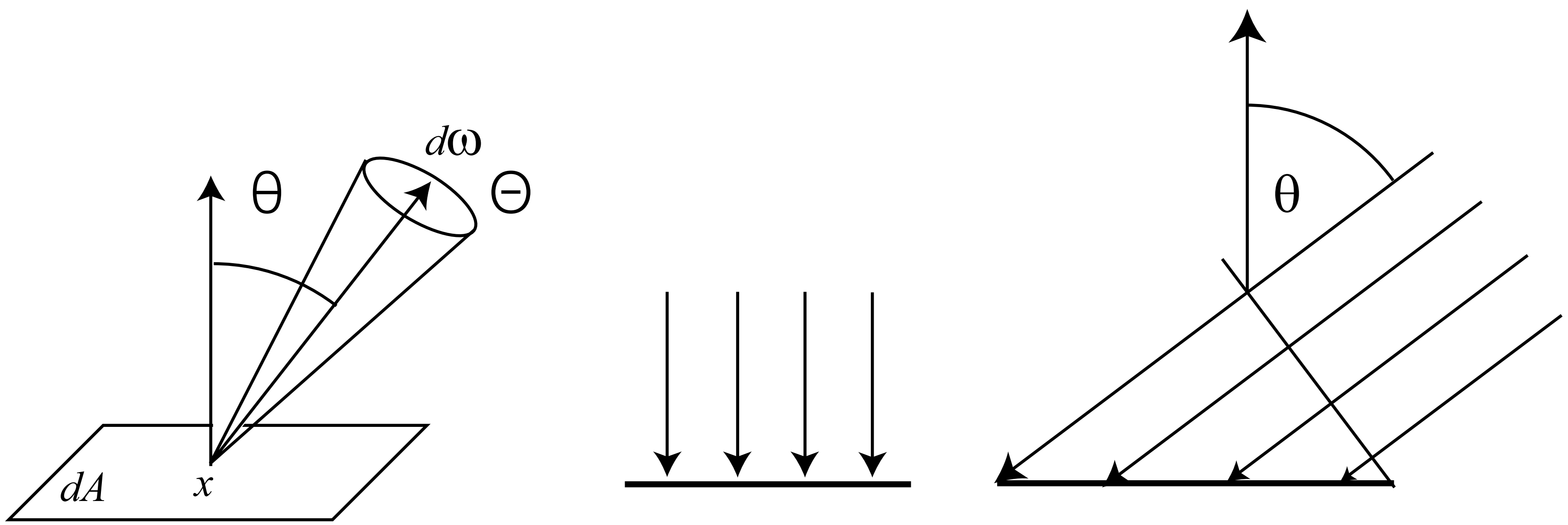
- Radiance is flux per unit projected area per unit solid angle.

$$L = \frac{d^2\Phi}{d\Omega\, dA^{\perp}} = \frac{d^2\Phi}{d\Omega\, dA \cos\theta}, [Wst^{-1}m^{-2}]$$

- Radiance expresses how much power arrives (or leaves from) a certain point on a surface, per unit solid angle and per unit projected area.
- Radiance is a 5-dimensional quantity that varies with position (3d) and direction (2d): $L(x,\boldsymbol{\theta})$

Radiance

- Radiance is the most important quantity in global illumination algorithms as it captures the power arriving at a point from a certain direction per unit projected area.



$$L = \frac{d^2\Phi}{d\Omega\, dA^{\perp}} = \frac{d^2\Phi}{d\Omega\, dA \cos\theta}, [Wst^{-1}m^{-2}]$$

Relationships

Flux: $$\Phi = \int_A \int_\Omega L(x \to \Theta) \cos\theta d\omega_\Theta dA_x$$

Irradiance: $$E(x) = \int_\Omega L(x \leftarrow \Theta) \cos\theta d\omega_\Theta$$

Radiance arriving at point x from the direction $\boldsymbol{\Theta}$

Radiosity: $$M(x) = \int_\Omega L(x \to \Theta) \cos\theta d\omega_\Theta$$

Radiance leaving at point x in the direction $\boldsymbol{\Theta}$

Radiance (incoming and outgoing)

- Radiance varies with position and direction:

$$L(x, \Theta) = \frac{d^2\Phi}{d\Omega\, dA \cos\theta}$$

- Radiance leaving point x along direction $\mathbf{\Theta}$:

$$L(x \rightarrow \Theta)$$

- Radiance arriving at point x from direction $\mathbf{\Theta}$:

$$L(x \leftarrow \Theta)$$

Radiance (wavelength dependency)

- Radiance also depends on wavelength:

$$L(x \to \Theta) = \int_{spectrum} L(x \to \Theta, \lambda) d\lambda$$

Radiance
$[Wst^{-1}m^{-2}]$

Spectral radiance
$[Wst^{-1}m^{-3}]$

It's this wavelength dependency that is responsible for colour

Properties of Radiance

1. Radiance is invariant along straight paths:

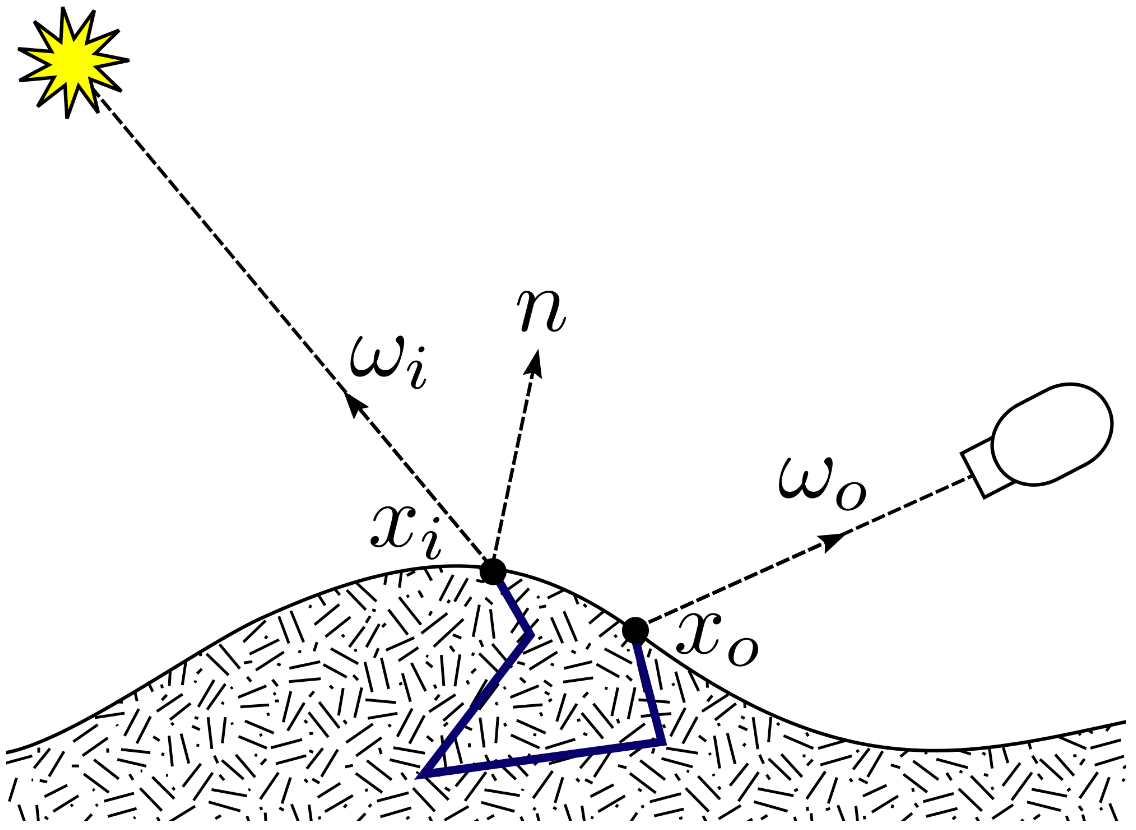
$$L(x \to y) = L(y \leftarrow x)$$

2. Sensors (cameras, human eye) are sensitive to radiance.

The two properties explain why perceived color or brightness does not change with distance.

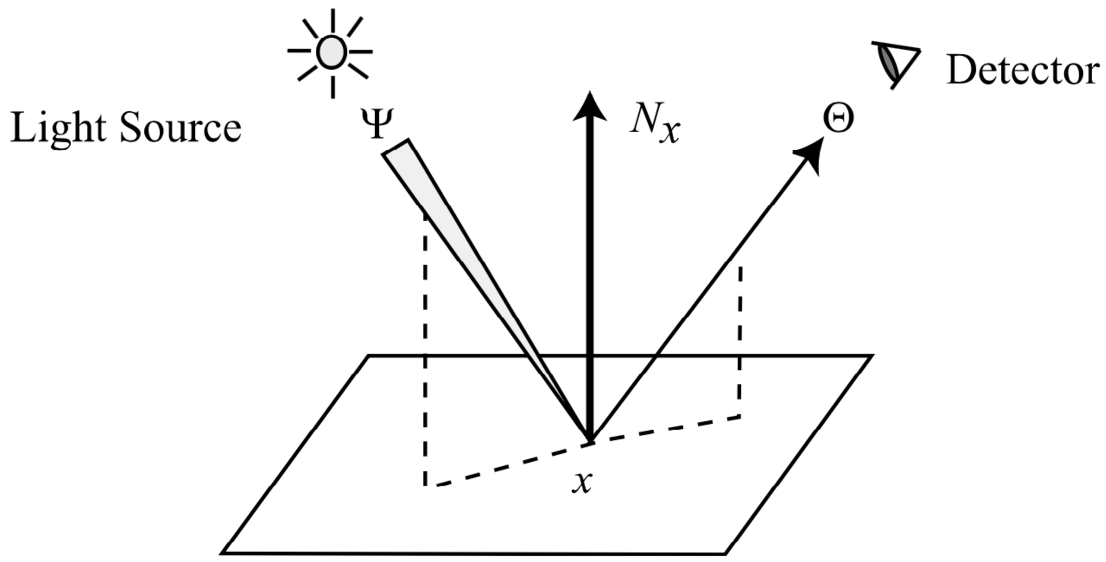
Scattering

- Light is scattered by objects in very complex and different ways
- In the general case light arriving at point x_i with wavelength λ_i and direction ω_i, can be scattered towards direction ω_o at point x_o with wavelength λ_o.
- If we keep the wavelength fixed, given an incoming direction and a location, there is a probability that light will exit at a given point with a given direction.
- This probability is given by the Bidirectional Surface Scattering Reflectance Distribution Function (BSSRDF)



BRDF

- If we stick to reflection and not allow for subsurface scattering then…
- … light will exit the surface at the same point x.
- There is a probability that light arriving at x along $\boldsymbol{\Psi}$ will exit along direction $\boldsymbol{\Theta}$.
- The function that takes into account that probability is called Bidirectional Reflectance Distribution Function (BRDF).

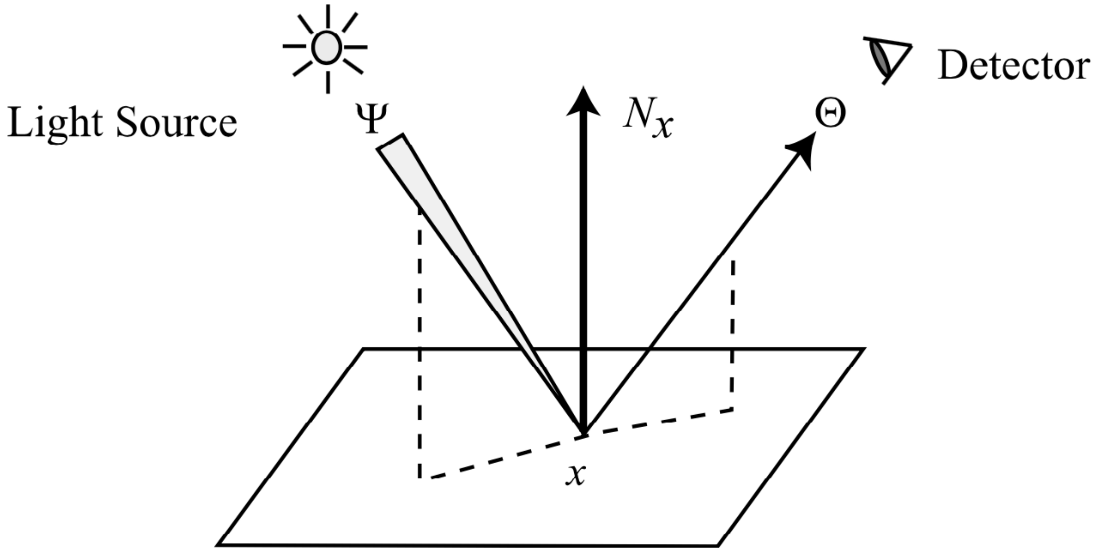


BRDF

- BRDF is defined as the ratio between differential radiance reflected towards Θ at a point x and the incoming irradiance incident through a differential solid angle $d\omega_\Psi$.

$$f_r(x, \Psi \to \Theta) = \frac{dL(x \to \Theta)}{dE(x \leftarrow \Psi)}$$

$$f_r(x, \Psi \to \Theta) = \frac{dL(x \to \Theta)}{L(x \leftarrow \Psi)\cos(N_x, \Psi)d\omega_\Psi}$$



BRDF Properties

- BRDF can take any positive value and vary with wavelength.
- It is a four dimensional function with two dimensions for each direction (in and out).
- Helmoltz Reciprocity: $f_r(x, \Psi \rightarrow \Theta) = f_r(x, \Theta \rightarrow \Psi)$
- Incident vs. Reflected Radiance:

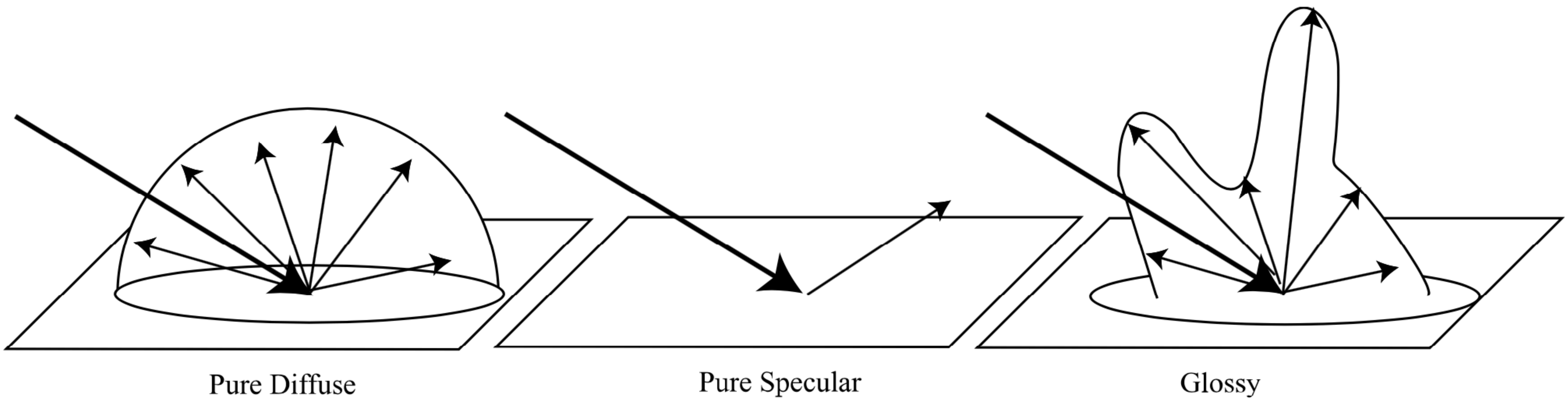
$$L(x \rightarrow \Theta) = \int_{\Omega_x} f_r(x, \Psi \rightarrow \Theta) L(x \leftarrow \Psi) \cos(N_x, \Psi) d\omega_\Psi$$

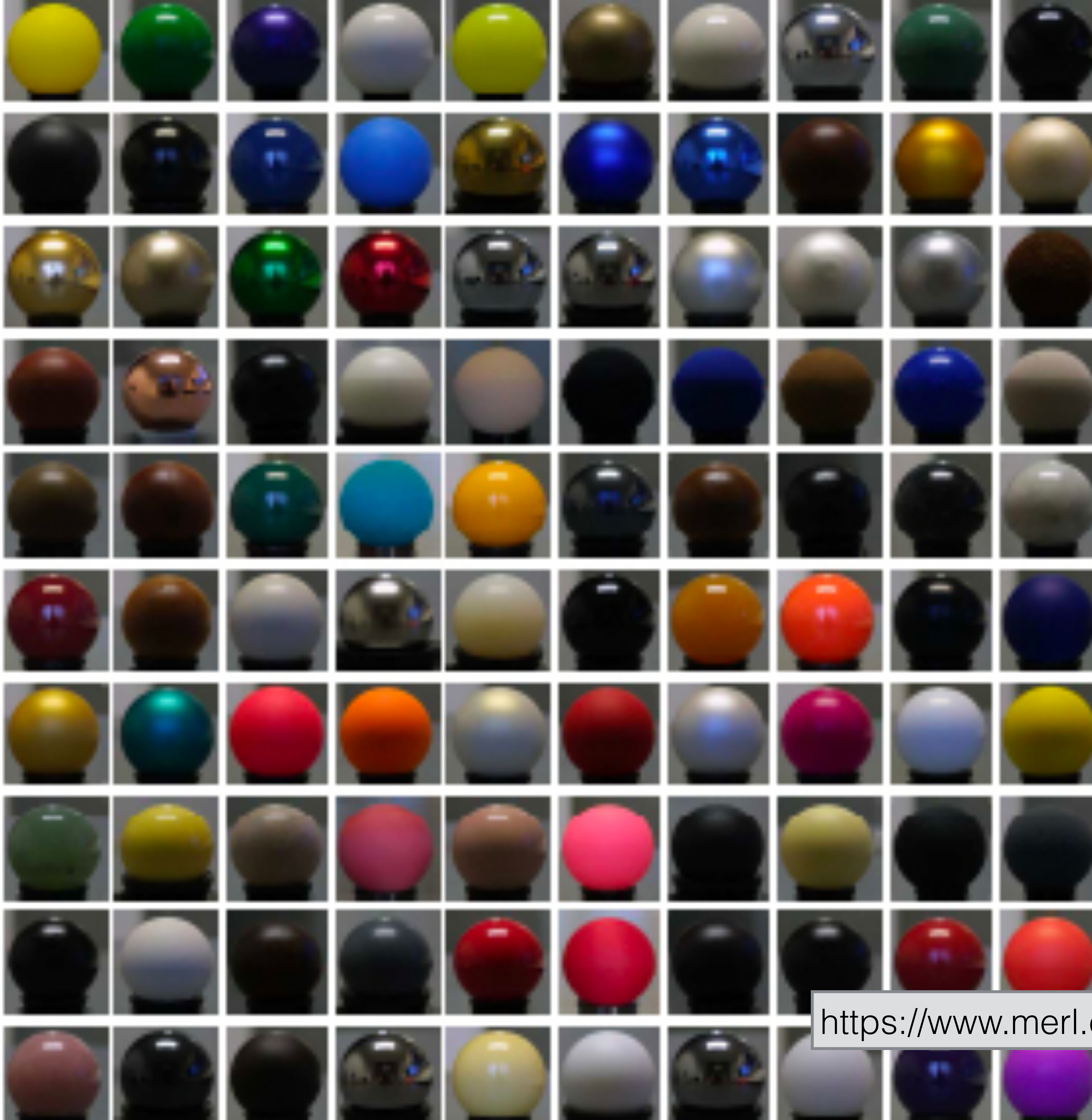
BRDF Properties

- Energy conservation…
- …total amount of power reflected over all directions must be less than or equal to the total amount of power incident on the surface.

$$\forall \Psi : \int_{\Omega_x} f_r(x, \Psi \rightarrow \Theta) \cos(N_x, \Theta) d\omega_\Theta \leq 1$$

BRDF Examples





https://www.merl.com/brdf/

Rendering Equation

$$L(x \to \Theta) = L_e(x \to \Theta) + L_r(x \to \Theta)$$

Emitted radiance

Reflected radiance

$$L_r(x \to \Theta) = \int_{\Omega_x} f_r(x, \Psi \to \Theta) L(x \leftarrow \Psi) \cos(N_x, \Psi) d\omega_\Psi$$

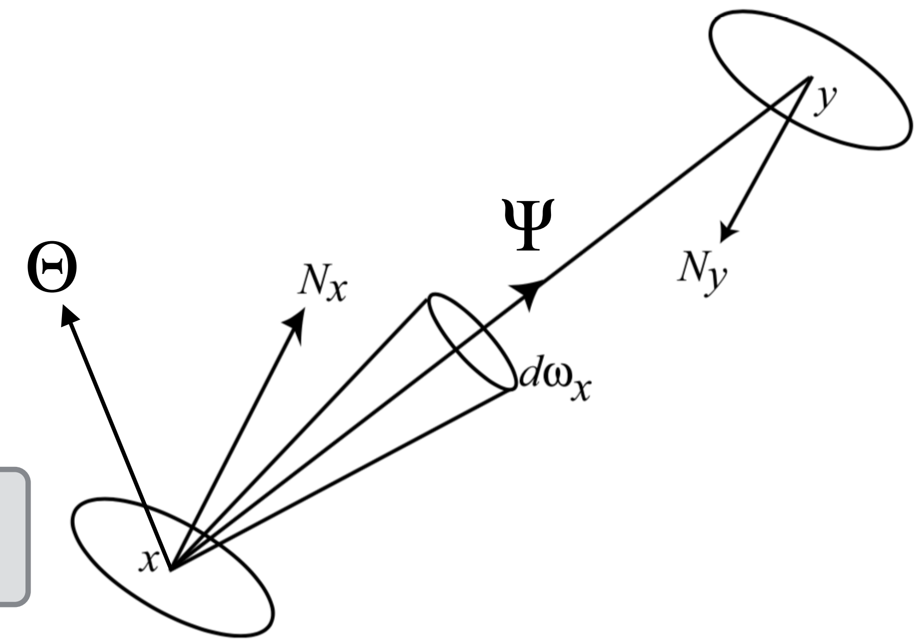
Hemispherical formulation

Rendering Equation

Emitted radiance

$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + L_r(x \rightarrow \Theta)$$

Reflected radiance



$$L_r(x \rightarrow \Theta) = \int_A f_r(x, \Psi \rightarrow \Theta) L(y \rightarrow -\Psi) V(x,y) G(x,y) dA_y$$

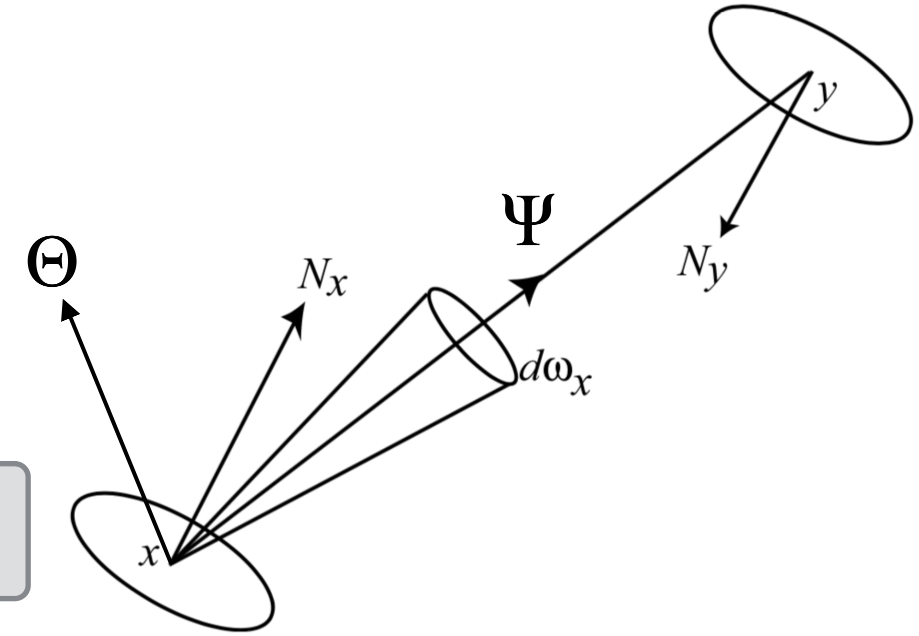
Area formulation

Rendering Equation

Emitted radiance

$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + L_r(x \rightarrow \Theta)$$

Reflected radiance



$$L_r(x \rightarrow \Theta) = \int_A f_r(x, \Psi \rightarrow \Theta) L(y \rightarrow -\Psi) V(x,y) G(x,y) dA_y$$

Area formulation

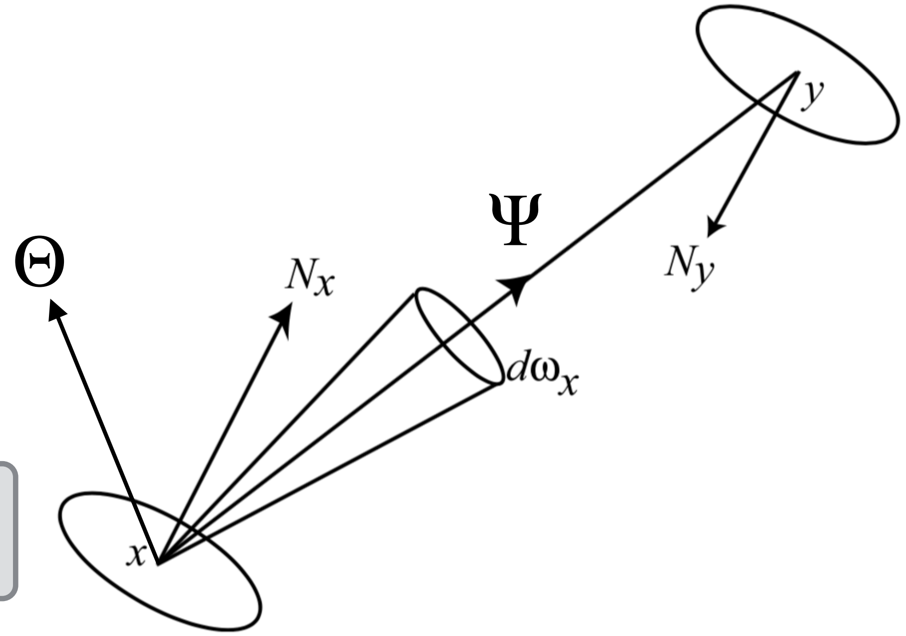
Visibility function:
1 if x and y are in line of sight
0 otherwise

Rendering Equation

Emitted radiance

$$L(x \to \Theta) = L_e(x \to \Theta) + L_r(x \to \Theta)$$

Reflected radiance



$$L_r(x \to \Theta) = \int_A f_r(x, \Psi \to \Theta) L(y \to -\Psi) V(x, y) G(x, y) dA_y$$

Area formulation

Geometrical Term:

$$\frac{\cos(N_x, \Psi)\cos(N_y, -\Psi)}{r_{xy}^2}$$

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