6.Lazy Learning

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Lazy learning

Summary

- Lazy Learning
- K-Nearest Neighbours
- Kernel Regression
- Kernel Density Estimation

Lazy Learning

Lazy vs Eager learning

- So far we saw examples of eager learning:

- Represent the hypothesis class with a model
- Train a model on the data, fitting parameters
- (Data can then be discarded)
- Answer based on the model

- With lazy learning there is no training step:

- No model to represent the hypothesis class
- No training; the data is simply stored
- Answer based on the data itself

K-Nearest Neighbours

K-NN

K-Nearest Neighbours classifier

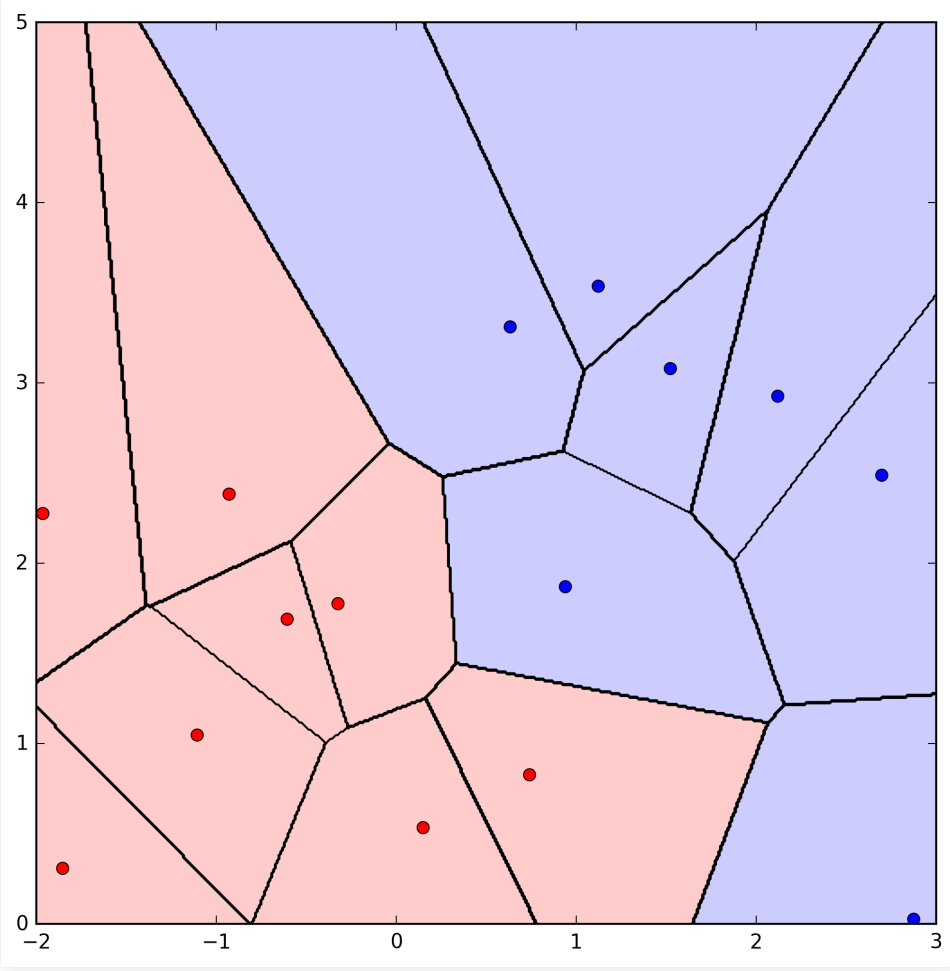
- Keep all the training set
- For new point, find closest k examples
- Return label of the majority

Lazy learning:

- No model to represent the hypothesis class
- No training; the data is simply stored
- Answer based on the data itself

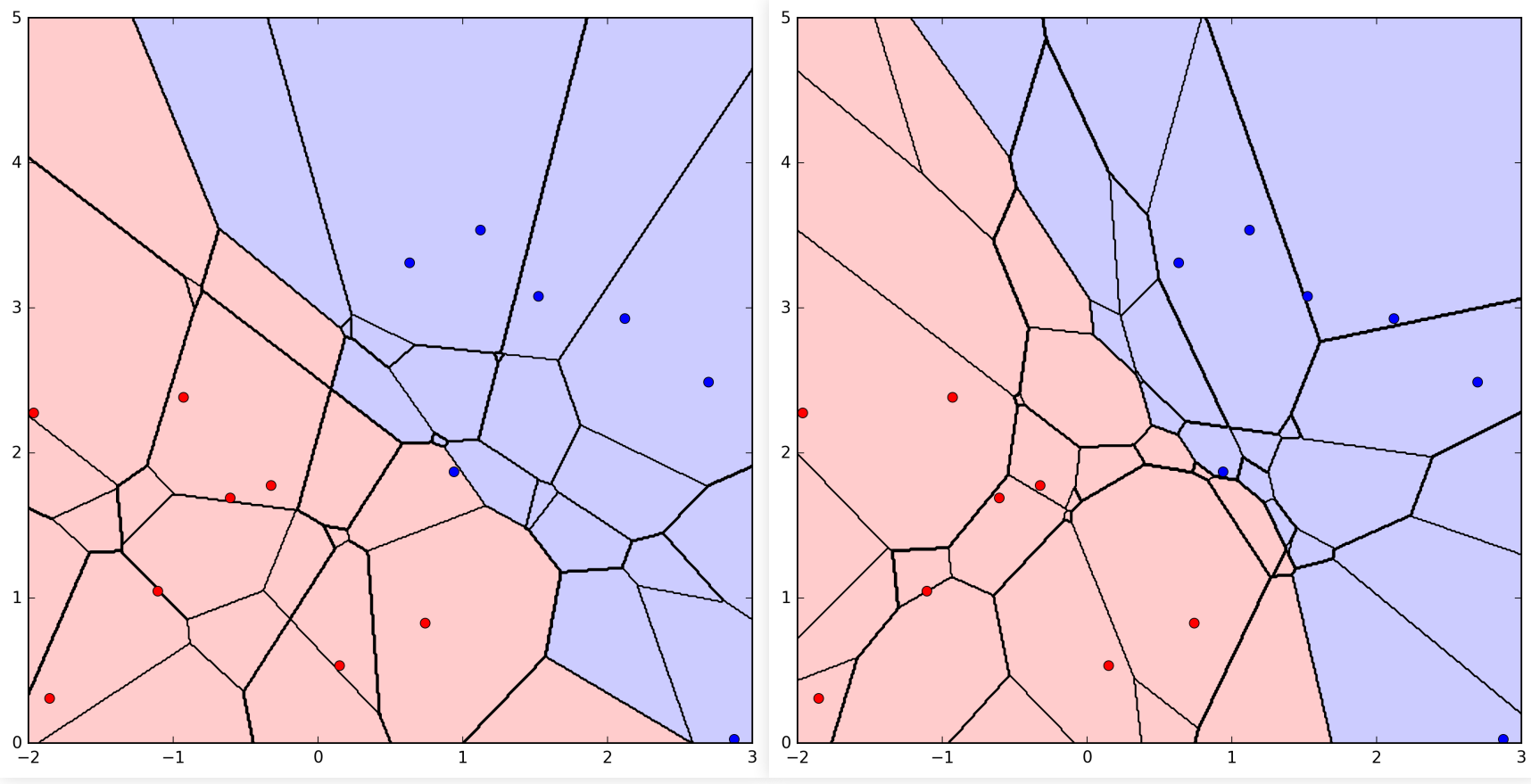
K-NN

- For k=1, the result is a Voronoy tesselation:



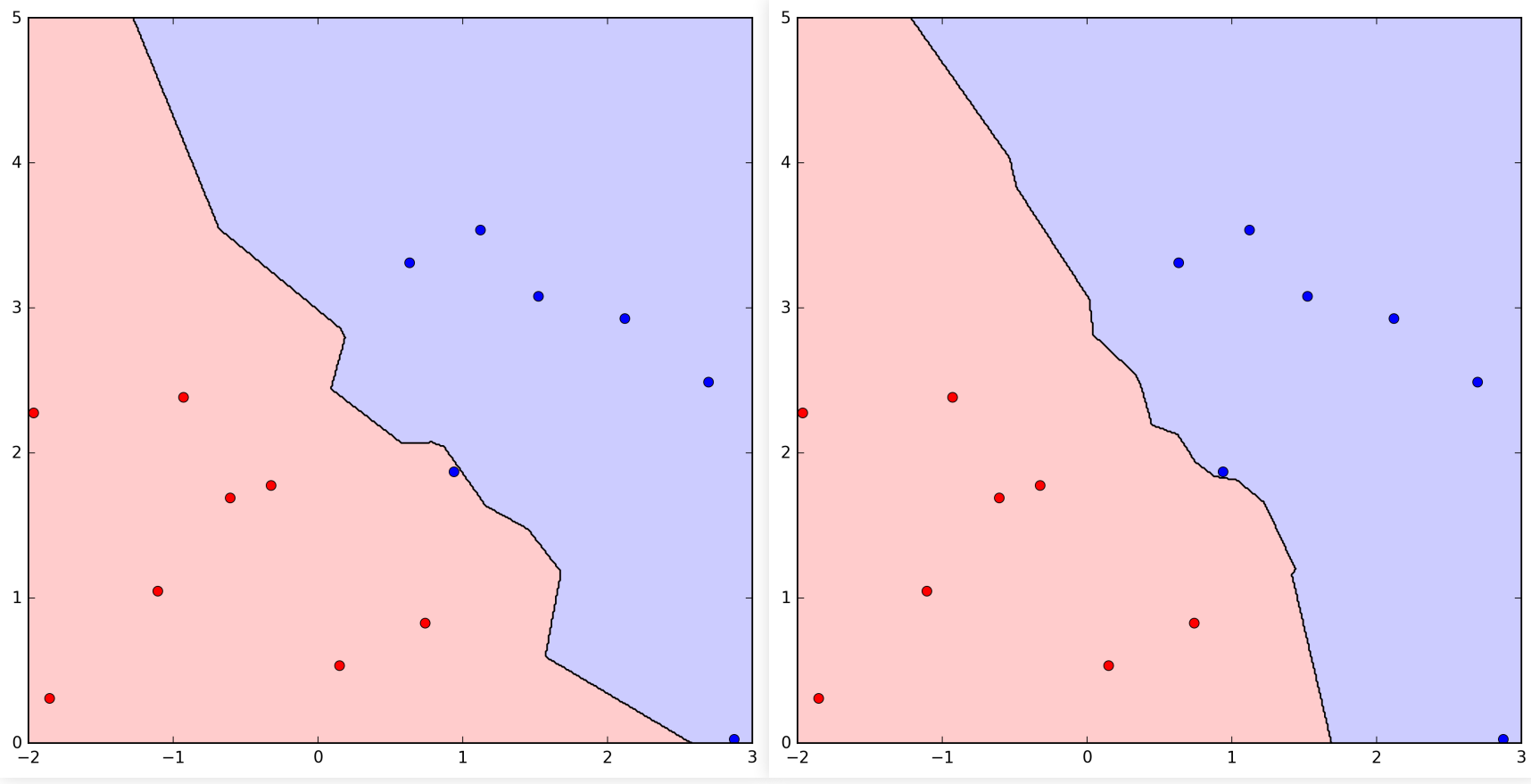
K-NN

- For larger values (3,5) more complex tesselation:



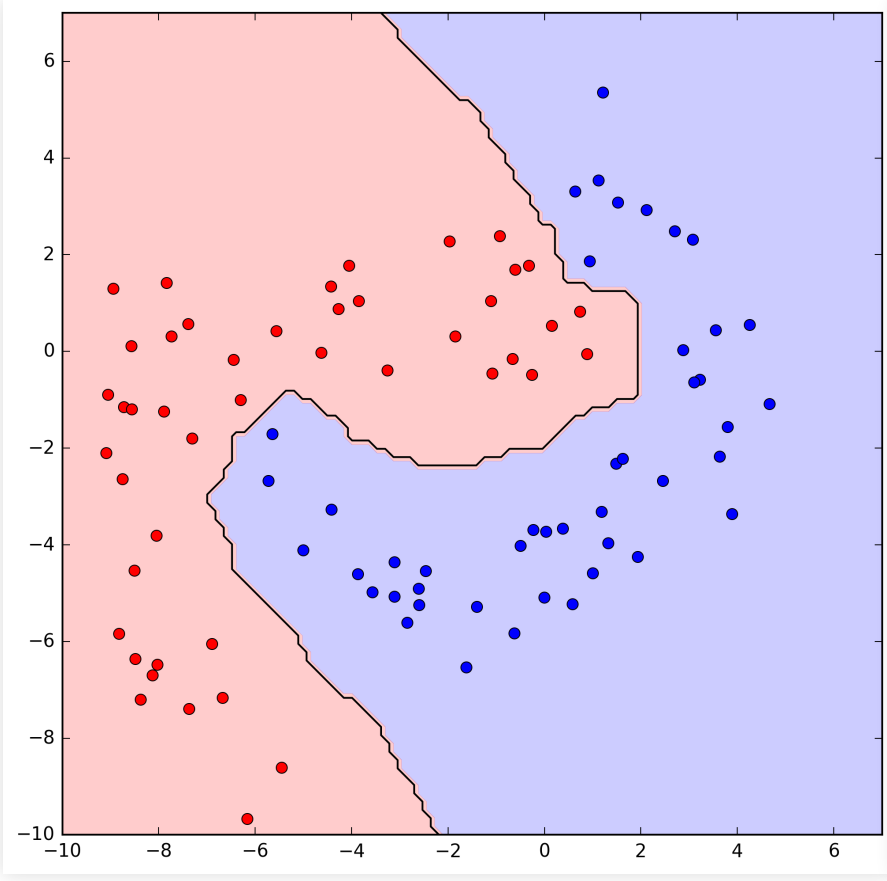
K-NN

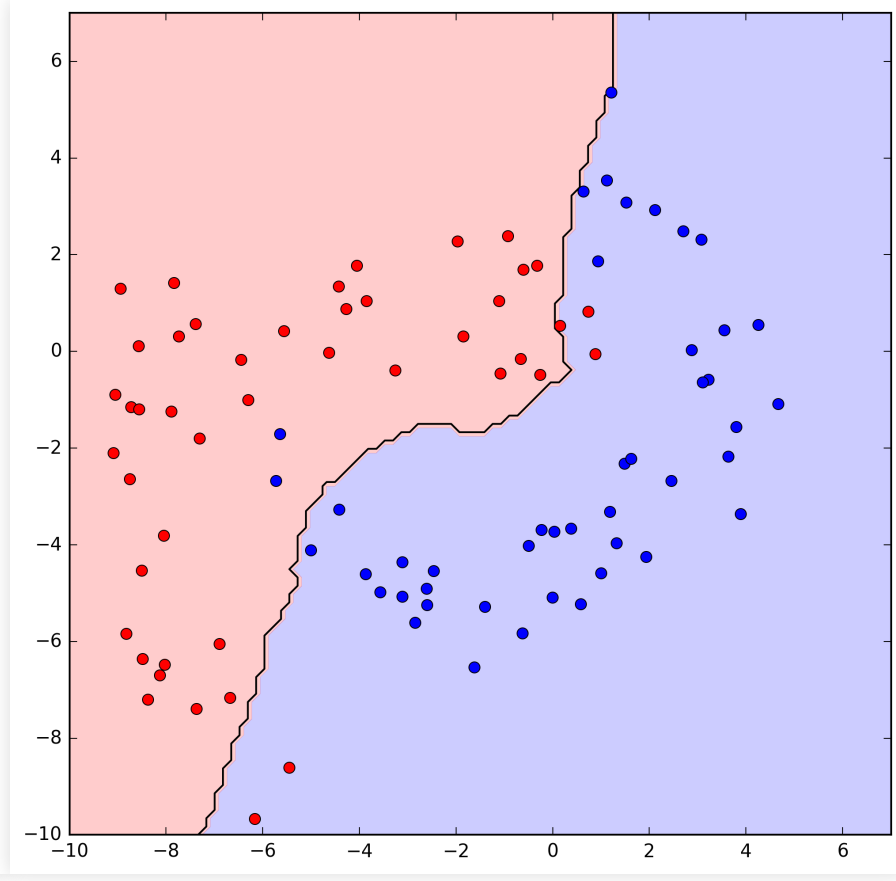
- In all cases, the decision boundary is piecewise linear



K-NN

- Higher k, less dependent on local conditions (1, 25)





Implement k-NN classifier

- First, we need a distance function
- For categorical data, Hamming distance

$$D_{x,x'} = \sum_d \begin{cases} 1, & x_d \neq x'_d \\ 0, & x_d = x'_d \end{cases}$$

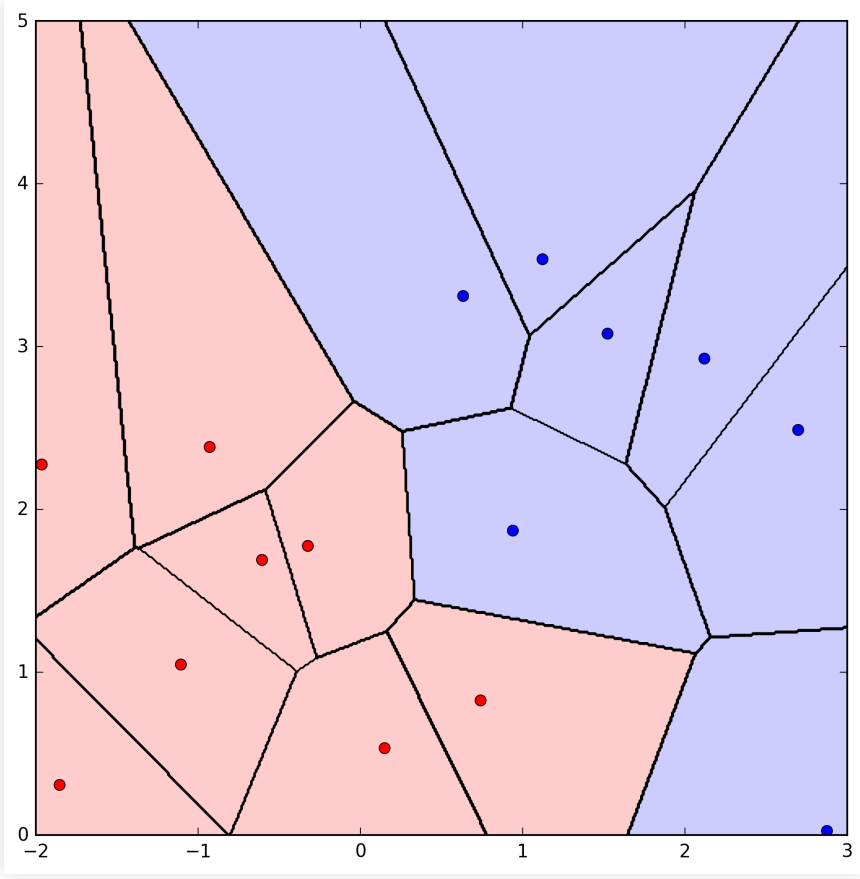
- For continuous data, Minkowski distance (or p-norm)

$$D_{x,x'} = \sqrt[p]{\sum_d |x_d - x'_d|^p}$$

- Manhattan $D_{x,x'} = \sum_d |x_d - x'_d|$
- Euclidean $D_{x,x'} = \sqrt{\sum_d |x_d - x'_d|^2}$

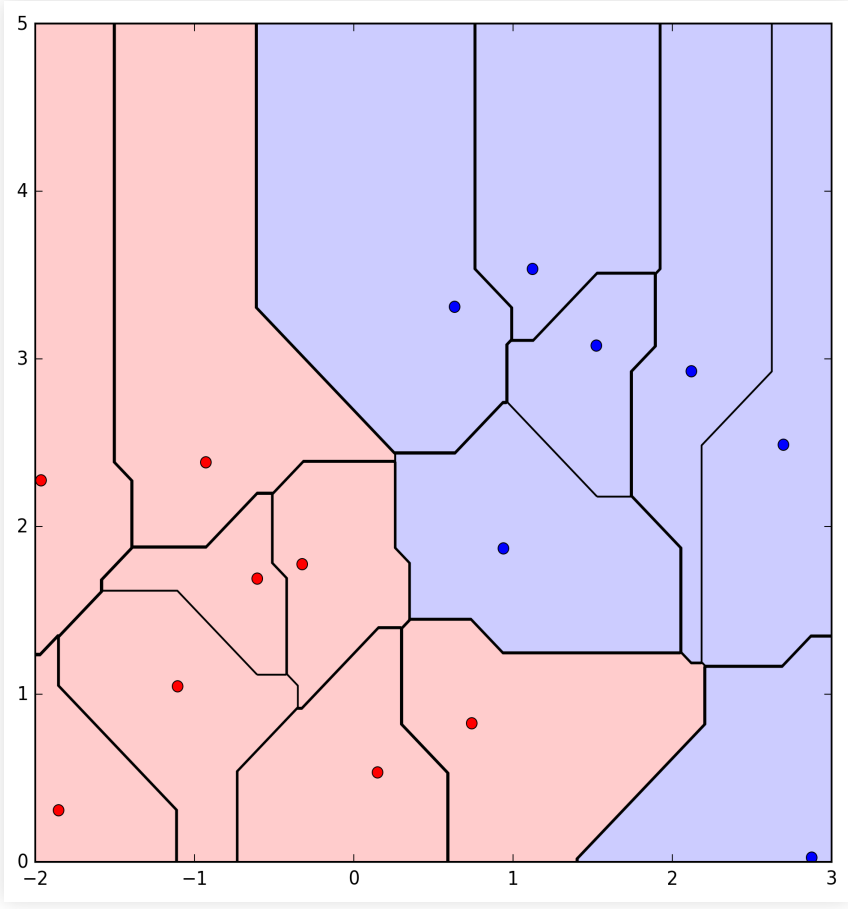
Implement k-NN classifier

- Minkowski distance, p=2 (Euclidean distance)

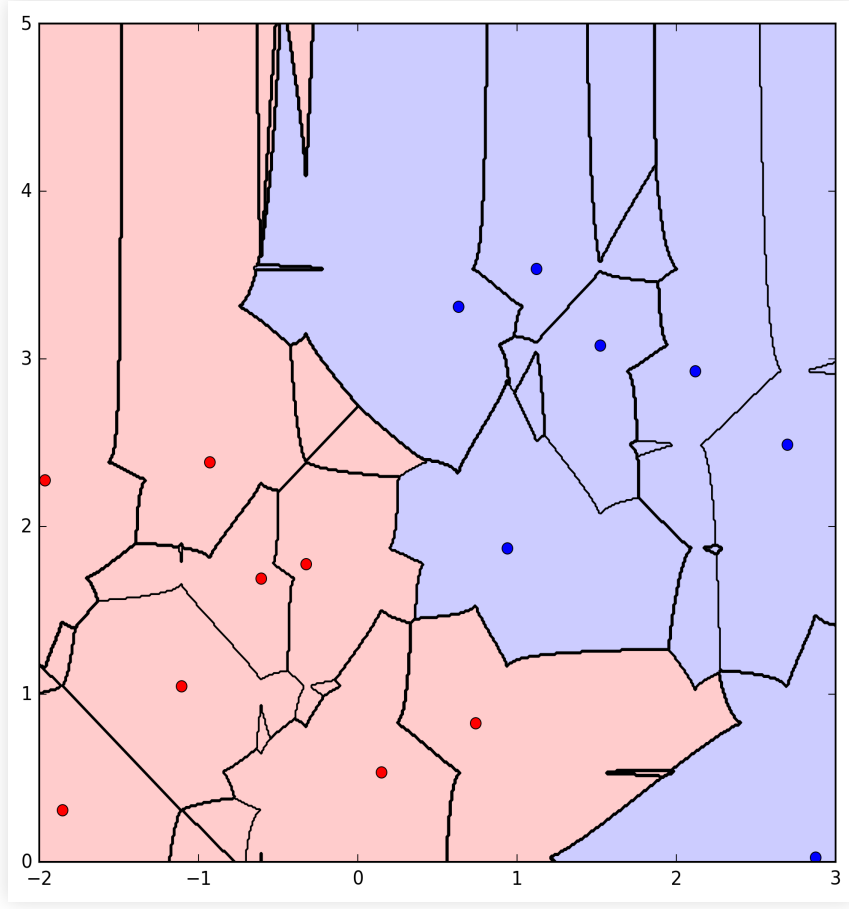


K-NN

- Minkowski, p=1



- Minkowski, p=0.7



Implement k-NN classifier

- Defining Minkowski distance (default p=2)

```
import numpy as np

def mink_dist(x, X, p = 2):
    """return p-norm values of point x distance to vector X"""
    sq_diff = np.power(np.abs(X - x),p)
    dists = np.power(np.sum(sq_diff,1),1.0/p)
    return dists
```

- Note: broadcasting, align by highest dimensions

See more on https://docs.scipy.org/doc/numpy-1.13.0/user/basics.broadcasting.html

Implement k-NN classifier

- Classify by majority vote

```
from scipy.stats import mode

def k_nearest_ixs(x, X,k):
    ixs = np.argsort(mink_dist(x,X))
    return ixs[:k]

def knn_classify(x,X,Y,k):
    ix = k_nearest_ixs(x,X,k)
    return mode(Y[ix,0], axis=None)[0][0]
```

- Note: mode returns two arrays, modes and counts.

See more on https://docs.scipy.org/doc/scipy/reference/generated/scipy.stats.mode.html

Implement k-NN classifier

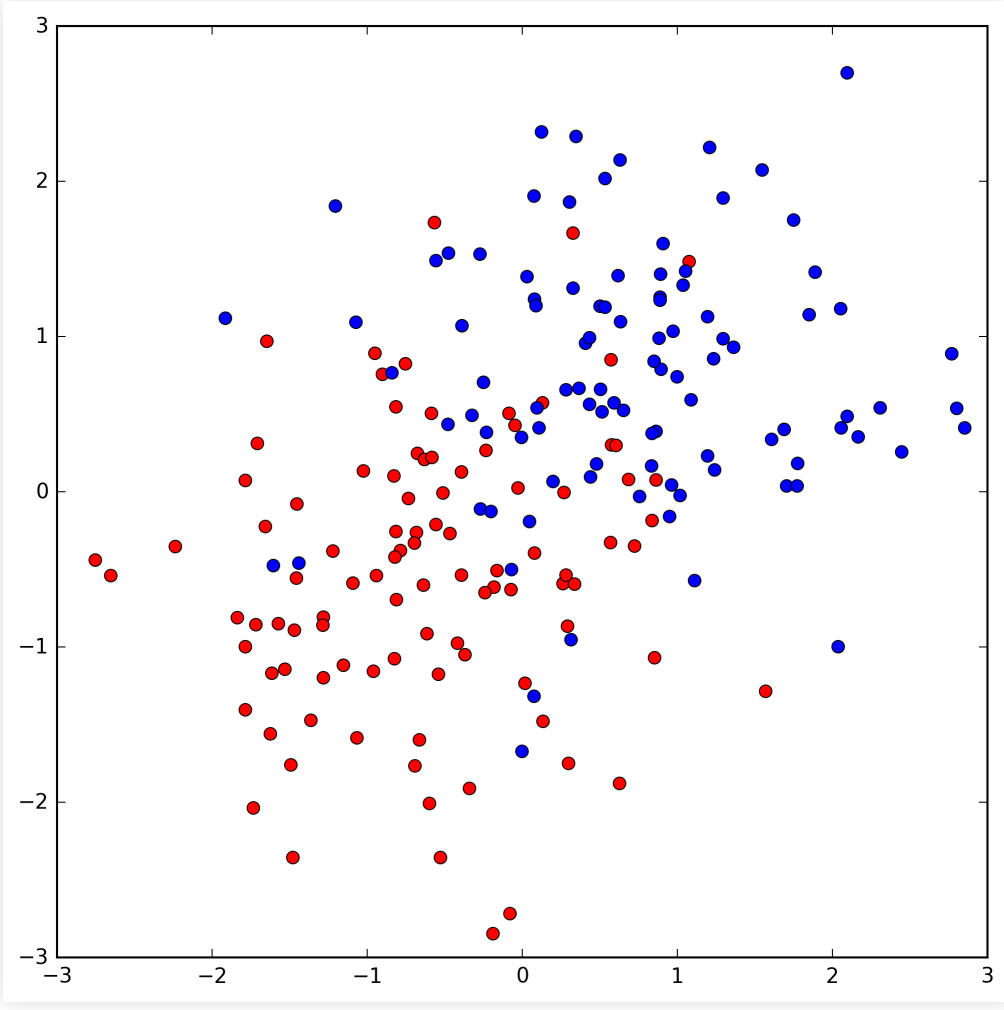
- Should we rescale the data?

- Distance functions are sensitive to scale
- However, care not to distort the data
- Depends on the problem (but generally, yes)

K-NN

Find the best k

- Load and set a third for tests



K-NN

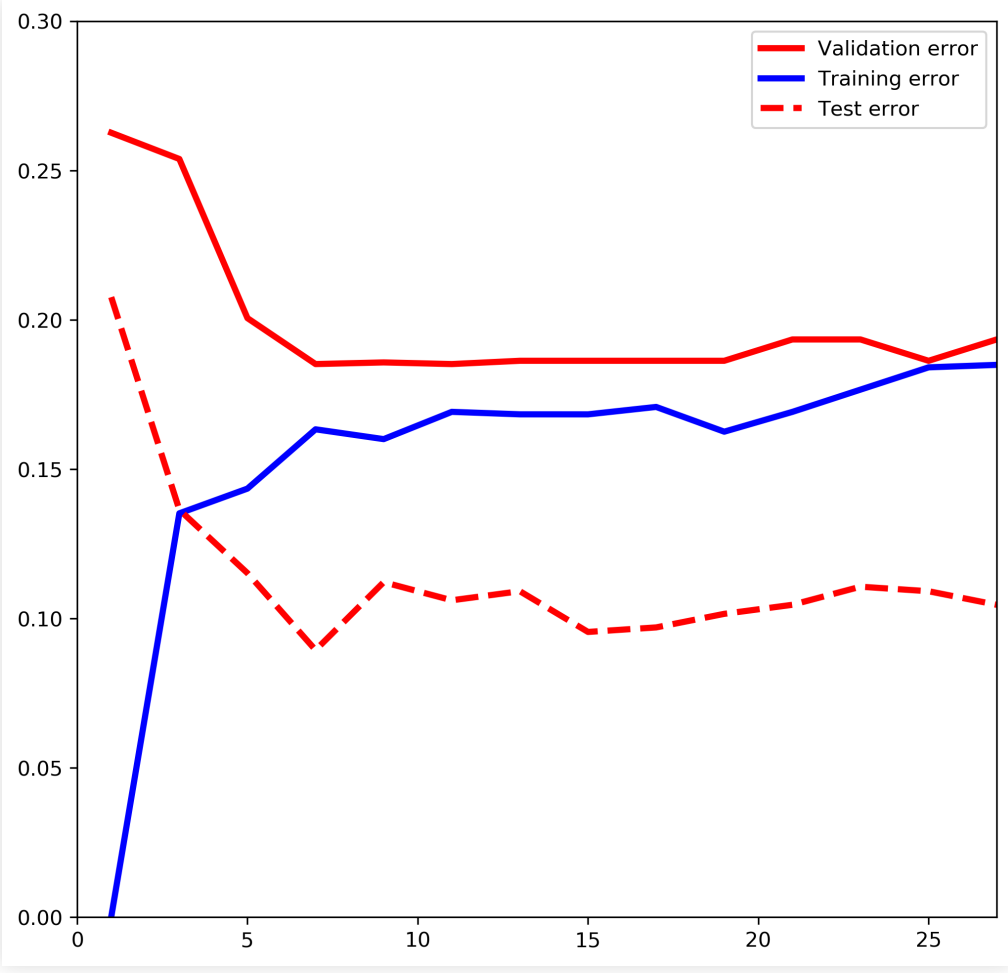
Find the best k

- Cross-validation, testing different values of k
- Only odd values of k, to avoid ties
- Error is $1 - accuracy$ (fraction of misclassifieds)

Important: cannot use test error

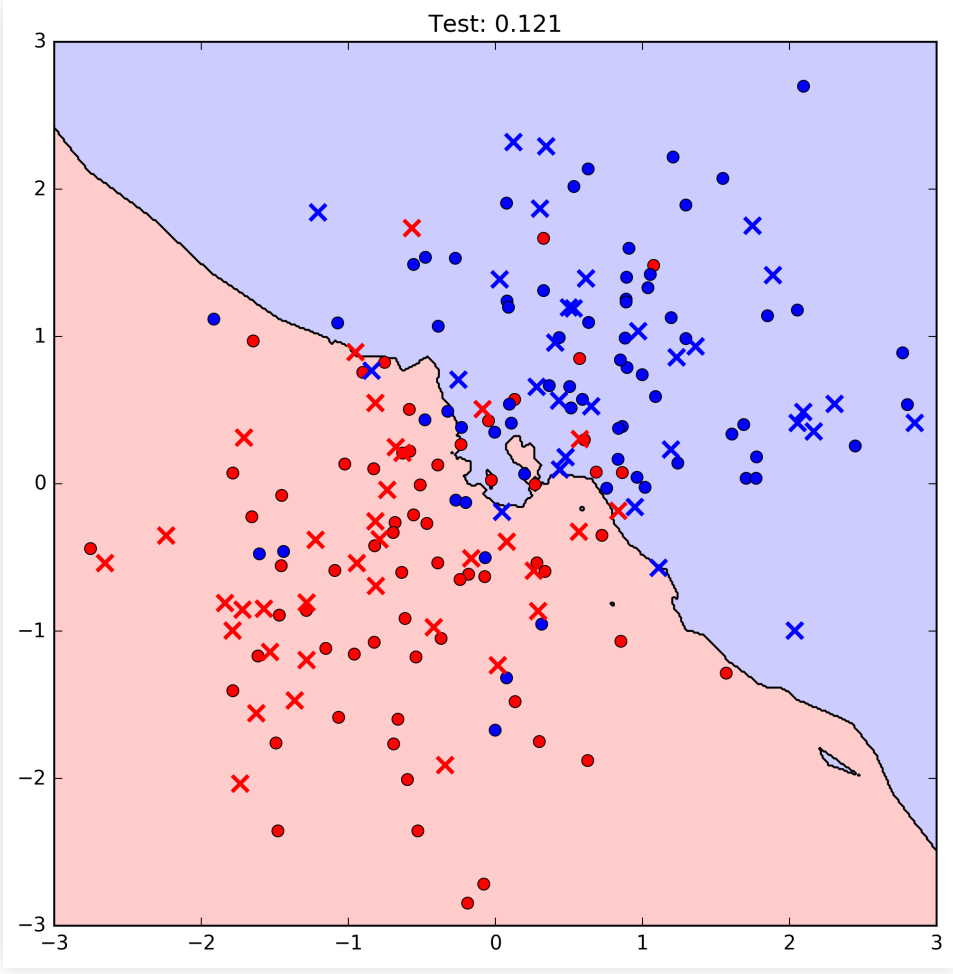
- If we do, then the test error becomes a biased estimator

Find the best k (Note error profile)



K-NN

- Use best k (k=9), train with full training set, test



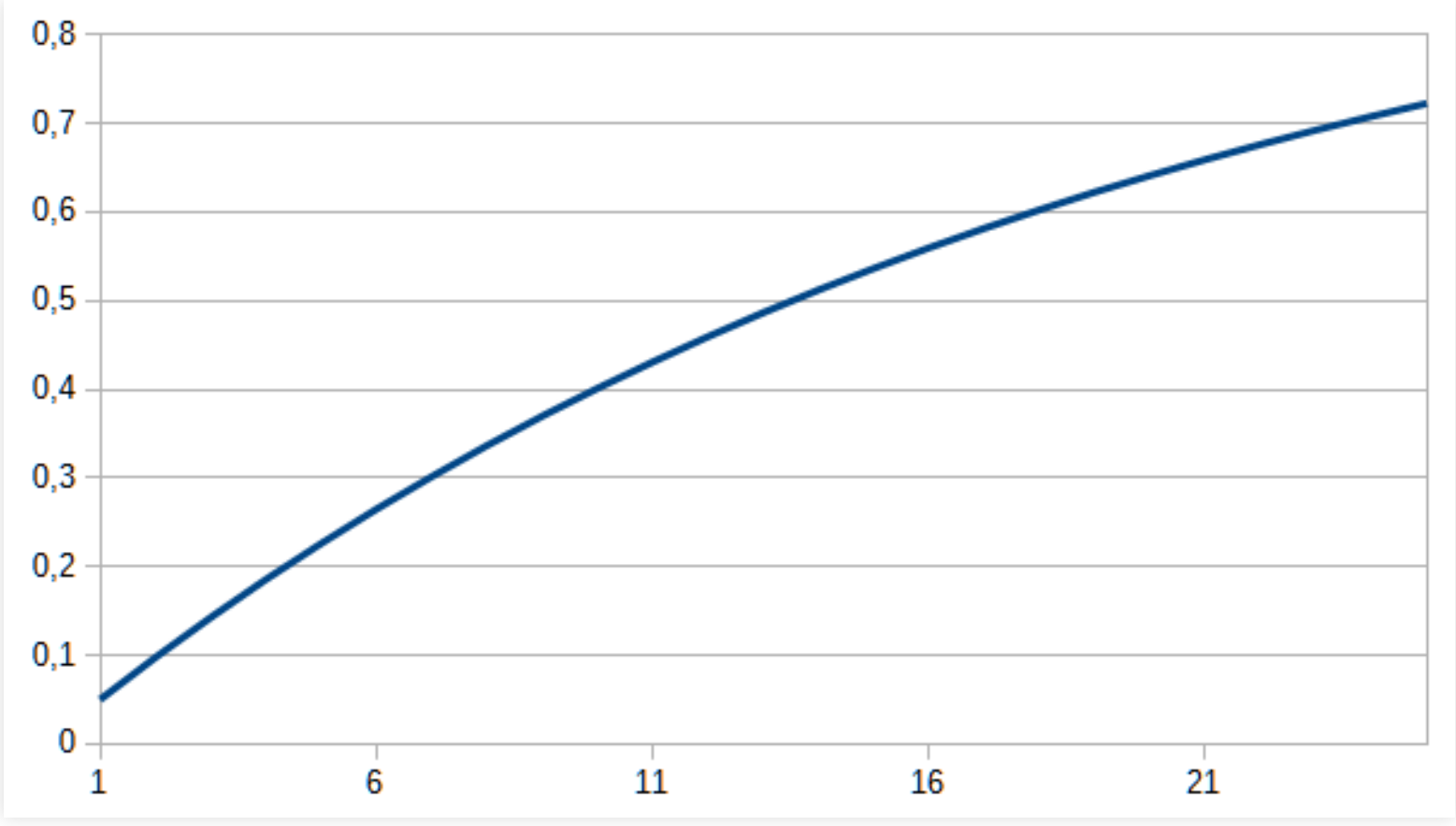
Curse of dimensionality

- Distance methods rely on defining regions inside some border.
- But with many dimensions this is a problem
- Imagine that border is 5% of diameter.

- In 1D border occupies 5% of region
- In 2D border occupies ~10%
- In 3D border occupies ~14%

Curse of dimensionality

- Distance methods work poorly in many dimensions



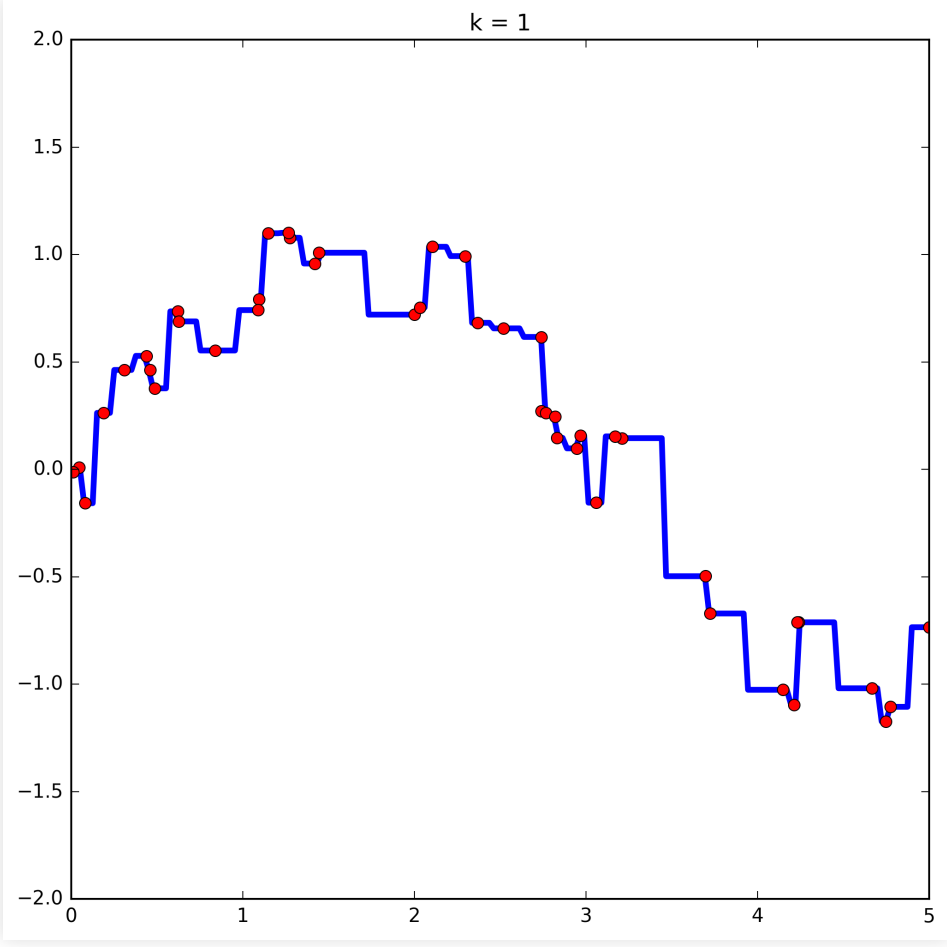
Regression

Regression

Instance Based Regression

- K-NN can be used for regression
- Predicted value: average of the k nearest neighbours

K-NN regression, k = 1

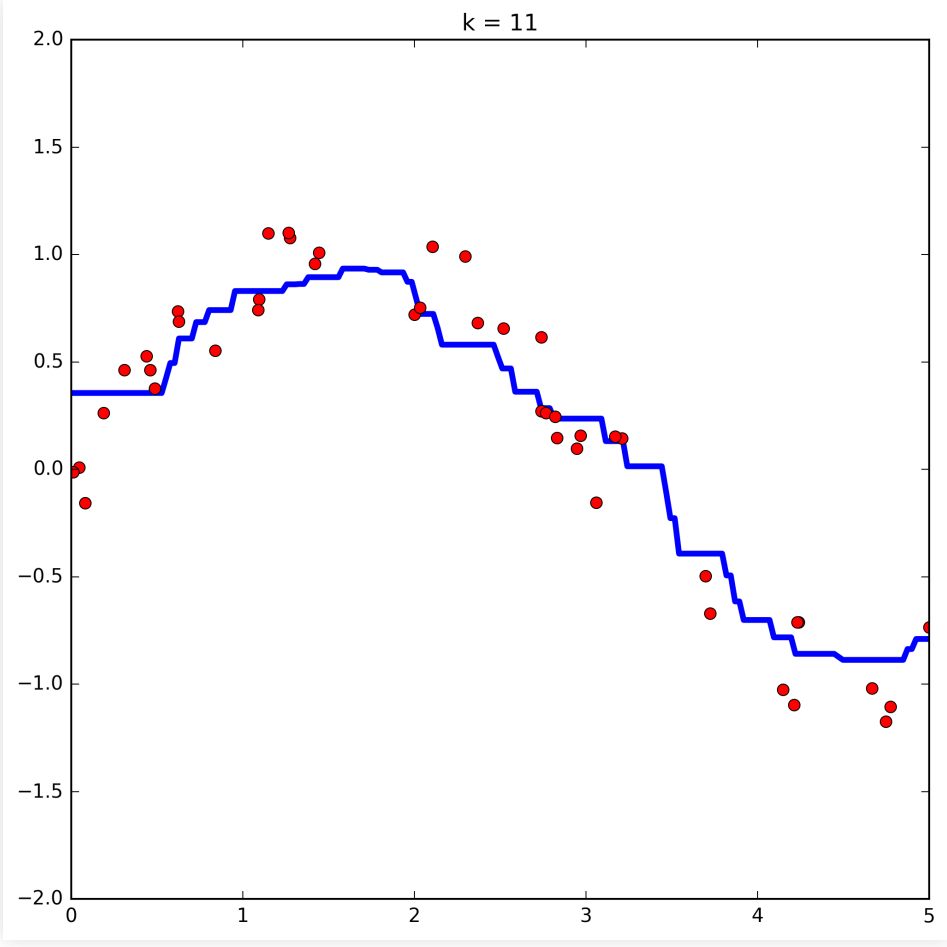


Regression

Instance Based Regression

- K-NN can be used for regression
- Predicted value: average of the k nearest neighbours

K-NN regression, k = 11

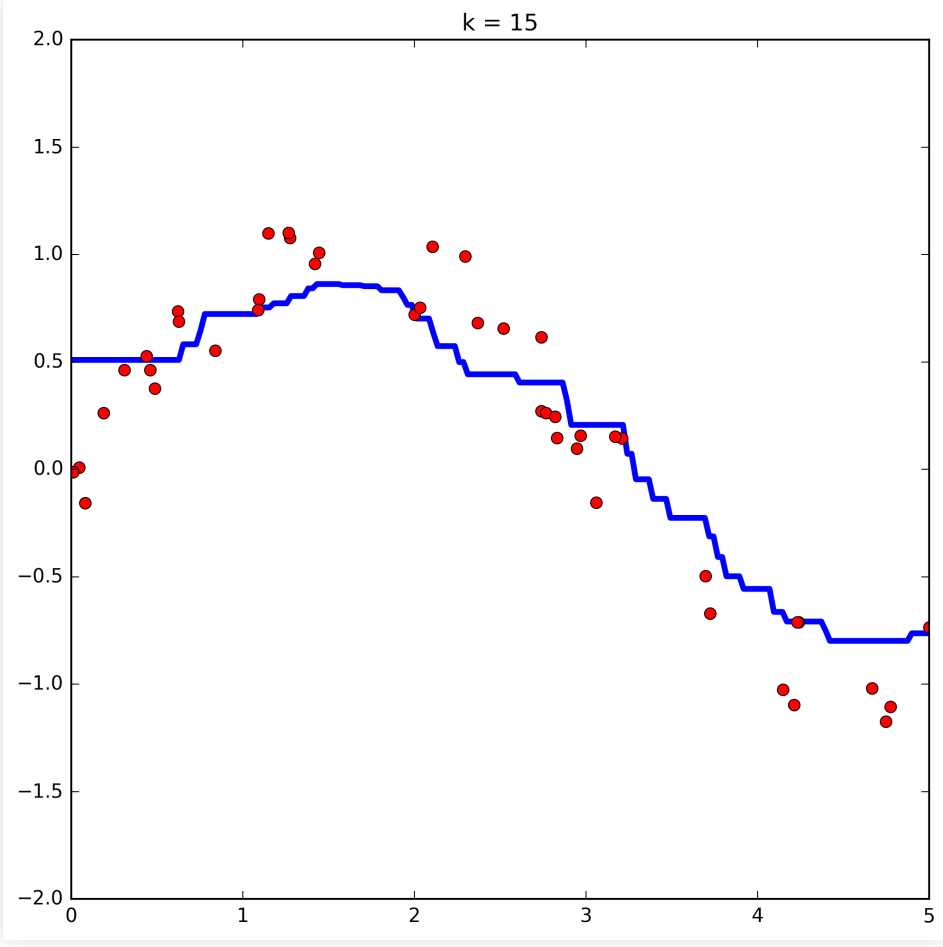


Regression

Instance Based Regression

- K-NN can be used for regression
- Predicted value: average of the k nearest neighbours

K-NN regression, k = 15



Instance Based Regression

- K-NN can be used for regression

- But discontinous prediction function
- We can solve this by giving less weight to more distant points, using all points
- Retains locality (closer weigh more), but a smooth curve

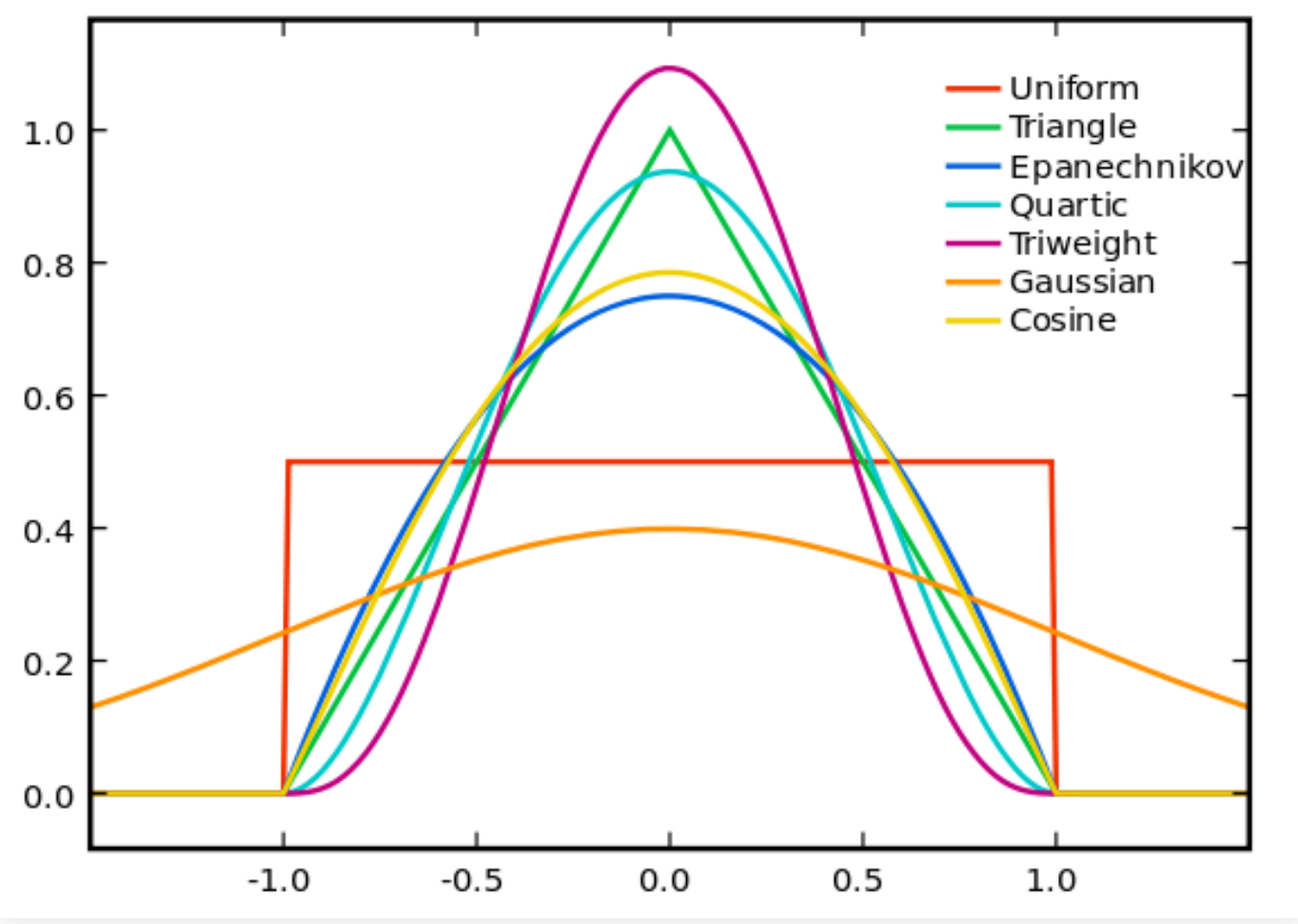
We'll use a **Kernel function**:

$$K(u) \geq 0 \ \ \forall u \qquad (non-negative)$$

$$\int_{-\infty}^{\infty} K(u)du = 1 \qquad (unitary\ area)$$

$$K(-u) = K(u) \ \ \forall u \qquad (symmetric)$$

Common Kernels



(source: Wikipedia)

Kernel regression, example: kernel and estimator

- Gaussian kernel:

$$K(u) = \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}}$$

- Nadaraya-Watson estimator

$$\hat{y}(x) = \frac{\sum_{t=1}^{N} K\left(\frac{x-x^t}{h}\right) y^t}{\sum_{t=1}^{N} K\left(\frac{x-x^t}{h}\right)}$$

- Or Priestley-Chao (for unequally spaced points):

$$\hat{y}(x) = \frac{1}{h} \sum_{t=2}^{N} (x^t - x^{t-1}) K\left(\frac{x - x^t}{h}\right) y^t$$

Implementing Kernel regression

- Gaussian kernel:

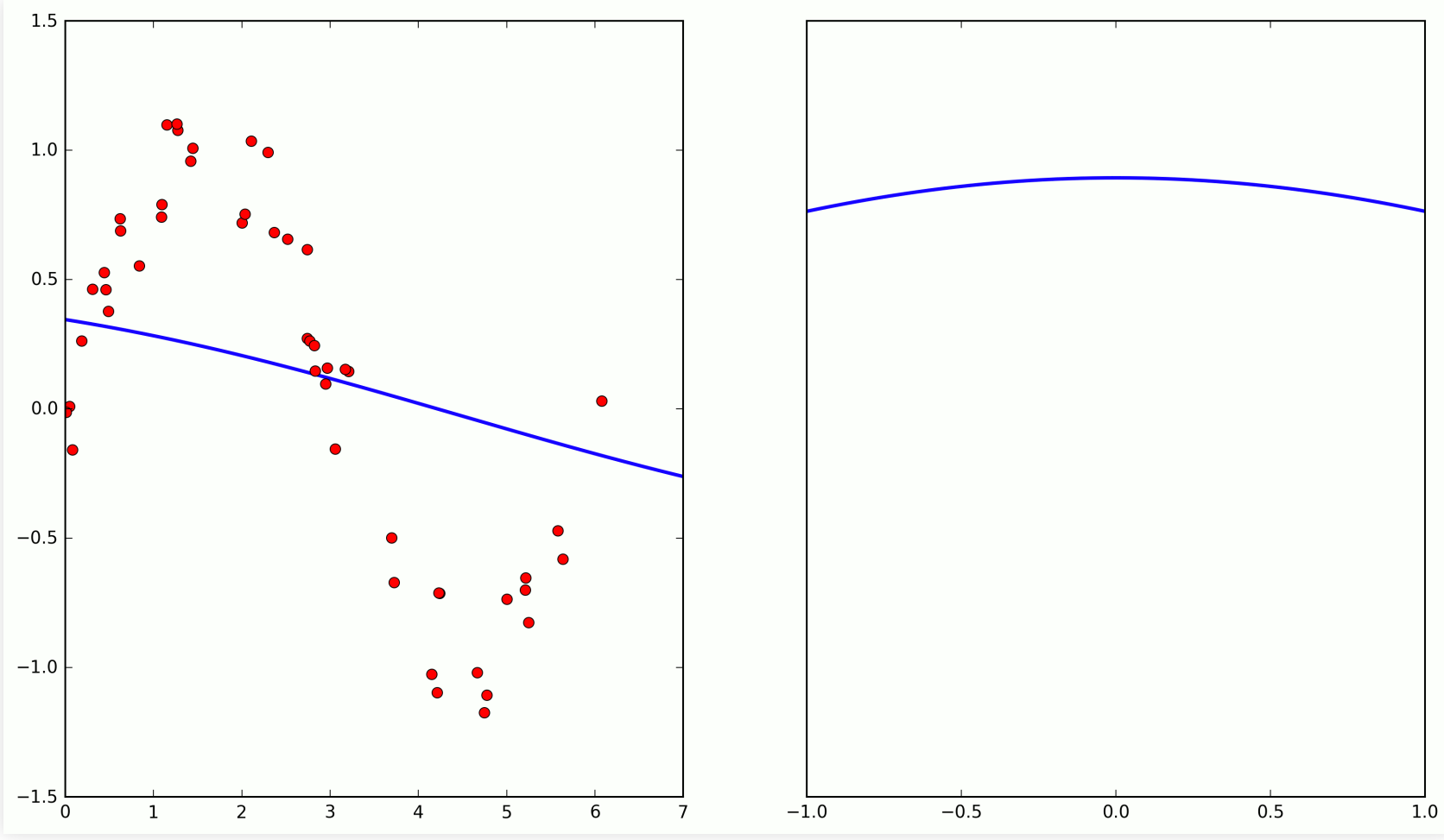
```
import numpy as np

def gaussiank(u):
  k=np.e**(-0.5*u**2)/np.sqrt(2*np.pi)
  return k
```

- Nadaraya-Watson estimator:

```
def nad_wat(K, h, X, Y, x):
    num = 0
    den = 0
    for ix in range(len(X)):
        u = (x-X[ix])/h
        k = K(u)
        num = num + Y[ix] * k
        den = den + k
    return num/den
```

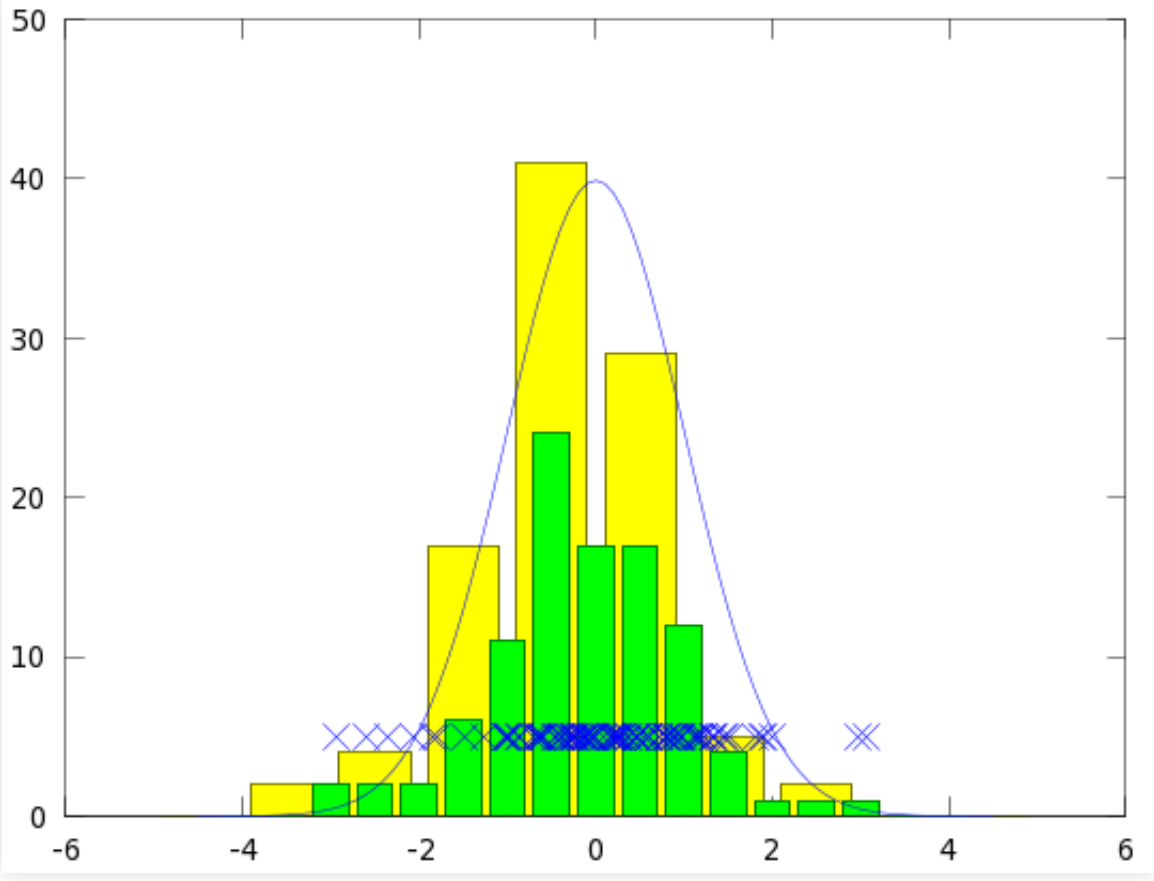
Regression

Curve depends on h



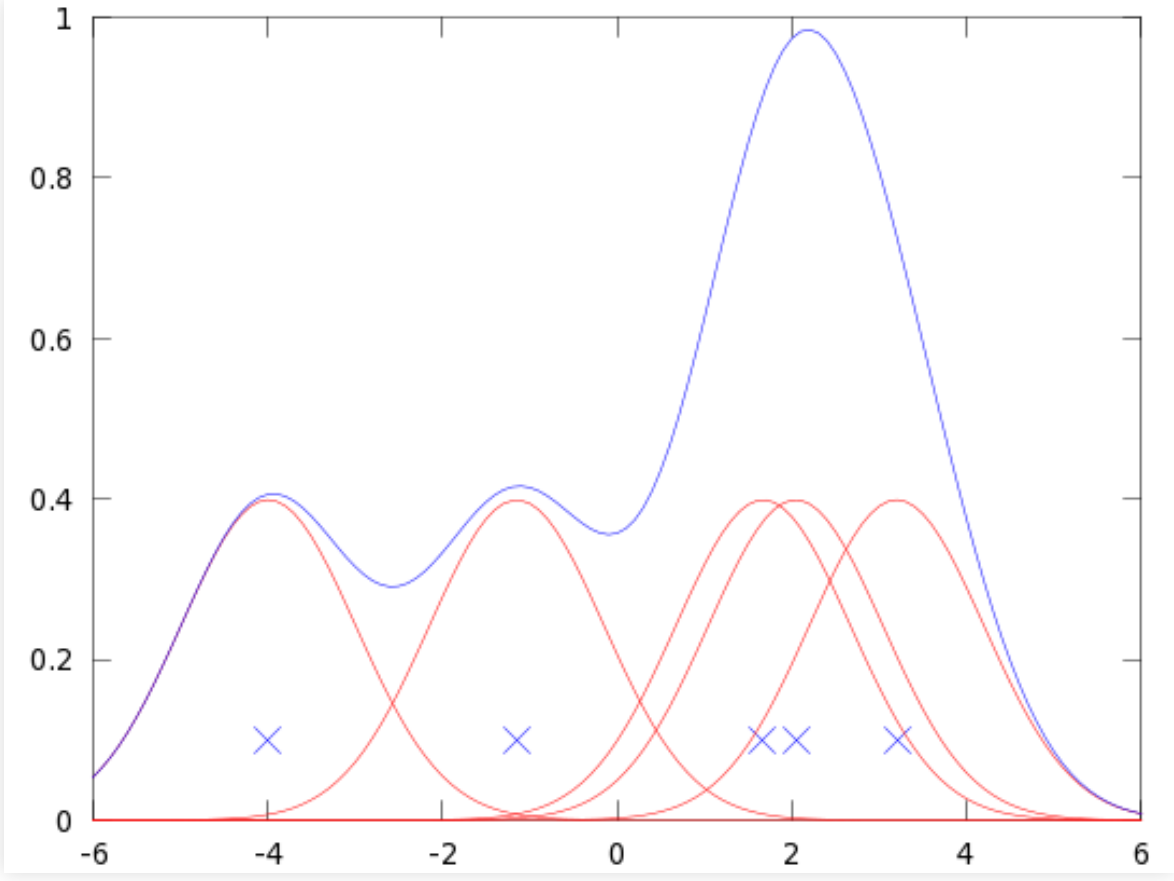
Kernel Density Estimation

- (Unsupervised learning): estimate distribution



Kernel Density Estimation

- Insted of histogram, add kernels



KDE, Scikit Learn

```
kde = KernelDensity(kernel='gaussian', bandwidth=bw)
kde.fit(X_r)
kde.score_samples(X_t)
```

- Note: score_samples returns the logarithm of the density (useful for probabilities)

Lazy Learning

- For KDE store the data points
- Compute function value based on the data

Regression

Regression

Summary

- Lazy Learning vs Eager Learning
- K-NN classification
- Curse of dimensionality
- K-NN regression
- Kernel regression and density estimation

Further reading

- Alpaydin, Sections 8.1 through 8.4
- Mitchell, Sections 8.1 and 8.2
- Marsland, Section 8.4.