Knowledge Representation and Reasoning

Exercises on First-Order Logic

1 Alpine Club

Formulate the following pieces of knowledge as sentences of first-order logic:

Tony, Mike and John belong to the Alpine Club. Every member of the Alpine Club who is not a skier is a mountain climber. Mountain climbers do not like rain, and anyone who does not like snow is not a skier. Mike dislikes whatever Tony likes, and likes whatever Tony dislikes. Tony likes rain and snow.

2 Reduction to CNF

Rewrite all sentences in $KB = \{(p \vee q) \rightarrow r, r \rightarrow s, p\}$ in conjunctive normal form, and present KB in clausal form.

3 Propositional Resolution

a) Show by resolution that the following set of clauses is inconsistent (derive the empty clause from it):

$$[A, B, C], [A, B, \neg C], [A, \neg B, C], [A, \neg B, \neg C]$$
$$[\neg A, B, C], [\neg A, B, \neg C], [\neg A, \neg B, C], [\neg A, \neg B, \neg C]$$

b) Show by resolution that the following sentence is inconsistent:

$$\neg\neg A \wedge (\neg A \vee ((\neg B \vee C) \wedge B)) \wedge \neg C$$

4 First-Order Resolution

Determine whether the following sentences are valid using resolution:

a) $\exists x \forall y \forall z \left((P\left(y\right) \rightarrow Q\left(z\right)) \rightarrow (P\left(x\right) \rightarrow Q\left(x\right))\right)$

b) $\exists x \left(P\left(x\right) \rightarrow \forall y \left(P\left(y\right)\right)\right)$

c) $\neg\exists x \forall y \left(E\left(x, y\right) \leftrightarrow \neg E\left(y, y\right)\right)$

Show by resolution that the following set of clauses is inconsistent.

d) $[P(x), P(f(x))], [\neg P(y), P(f(z))], [\neg P(w), \neg P(f(w))]$

5 Alpine Club and First-Order Resolution

As a follow up to the Alpine Club Exercise, use resolution to prove that there exists a member of the Alpine club who is a climber but not a skier. Can you determine his name?