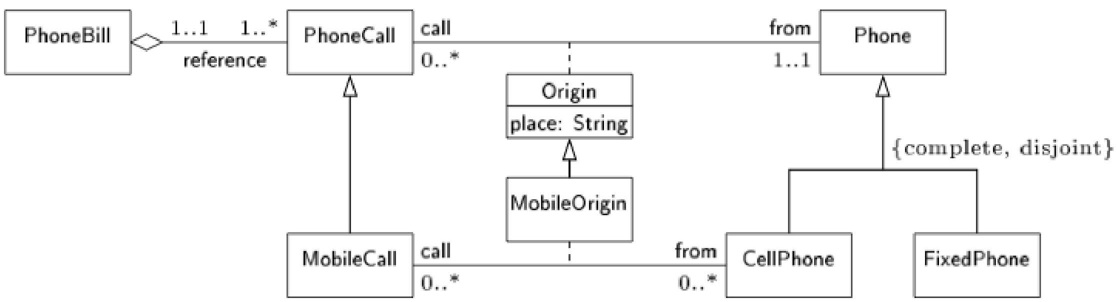
Knowledge Representation and Reasoning

Exercises on Description Logic Ontologies

1 Converting from Description Logics to First-Order Logic

Consider the solutions from the previous class on converting the following UML class diagram into description logics.



Convert the Description Logic result into first-order logic.

2 Constructing Models of Ontologies

Consider the following **TBox**:

$$
\begin{aligned}
&Cow \sqsubseteq Vegetarian\\
&MadCow \sqsubseteq Cow \sqcap \exists eat.BrainOfSheep\\
&Sheep \sqsubseteq Animal\\
&Vegetarian \sqsubseteq (\geq 1\ eat) \sqcap \forall eat.\neg(Animal \sqcup PartOfAnimal)\\
&BrainOfSheep \sqsubseteq PartOfAnimal
\end{aligned}
$$

1. Translate the TBox into natural language, and compare with the translation into first-order logic.
2. Construct a model for the ontology $\mathcal{O}_1 = (\mathbf{TBox}, \{Cow(mimosa)\})$.
3. Show that there is no model for the ontology $\mathcal{O}_2 = (\mathbf{TBox}, \{MadCow(mimosa)\})$.

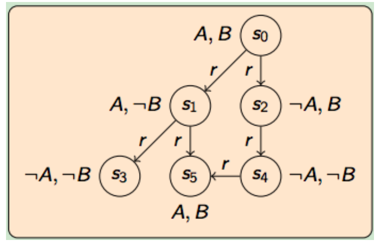
3 Knowledge Representation in $\mathcal{ALC}$

Express the following sentences in terms of the description logic $\mathcal{ALC}$.

1. All employees are humans.
2. A mother is a female who has a child.
3. A parent is a mother or a father.
4. A grandmother is a mother who has a child who is a parent.
5. Only humans have children that are humans.

4 Semantics of $\mathcal{ALC}$

Let $\mathcal{I}$ be the following $\mathcal{ALC}$ interpretation on the domain $\Delta^{\mathcal{I}} = \{s_0, s_1, ..., s_5\}$.

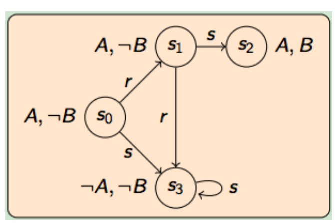


Determine the interpretation of the following concepts:

1. $\top^{\mathcal{I}}$.
2. $\perp^{\mathcal{I}}$.
3. $A^{\mathcal{I}}$.
4. $B^{\mathcal{I}}$.
5. $(A \sqcap B)^{\mathcal{I}}$.
6. $(A \sqcup B)^{\mathcal{I}}$.
7. $(\neg A)^{\mathcal{I}}$.
8. $(\exists r.A)^{\mathcal{I}}$.
9. $(\forall r.\neg B)^{\mathcal{I}}$.
10. $(\forall r.(A \sqcup B))^{\mathcal{I}}$.

5 Semantics of $\mathcal{ALC}$

Let $\mathcal{I}$ be the following $\mathcal{ALC}$ interpretation on the domain $\Delta^{\mathcal{I}} = \{s_0, s_1, ..., s_3\}$.



Determine the interpretation of the following concepts:

1. $(A \sqcup B)^{\mathcal{I}}$.
2. $(\exists s.\neg A)^{\mathcal{I}}$.
3. $(\forall s.A)^{\mathcal{I}}$.
4. $(\exists s.\exists s.\exists s.\exists s.A)^{\mathcal{I}}$.
5. $(\neg \exists r.(\neg A \sqcup \neg B))^{\mathcal{I}}$.
6. $(\exists s.(A \sqcup \forall s.\neg B) \sqcup \neg \forall r.\exists r.(A \sqcup \neg A))^{\mathcal{I}}$.

6 (Un)Satisfiability and Validity of $\mathcal{ALC}$

For each of the following formulas, indicate if it is valid, satisfiable or unsatisfiable. If it is not valid, provide a model that falsifies it:

1. $\forall r.\,(A \sqcap B) \equiv \forall r.A \sqcap \forall r.B$.
2. $\forall r.\,(A \sqcup B) \equiv \forall r.A \sqcup \forall r.B$.
3. $\exists r.\,(A \sqcap B) \equiv \exists r.A \sqcap \exists r.B$.
4. $\exists r.\,(A \sqcup B) \equiv \exists r.A \sqcup \exists r.B$.