Knowledge Representation and Reasoning

Exercises on Tableaux in Description Logics

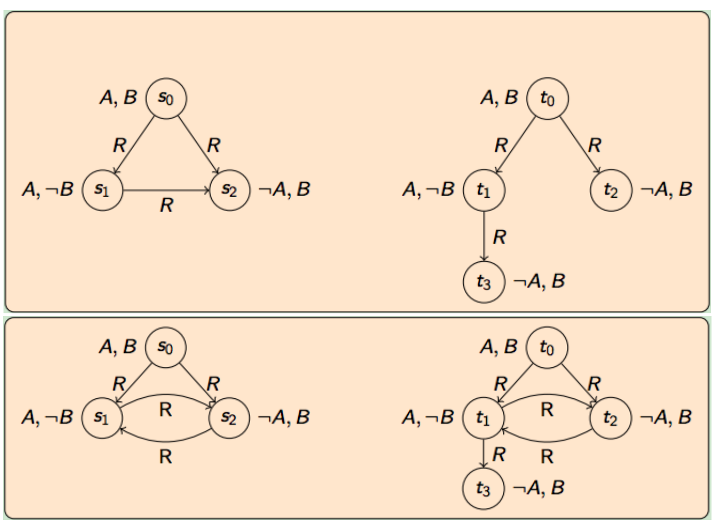
1 (Un)Satisfiability and Validity of $\mathcal{ALC}$

For each of the following concepts, indicate if it is valid, satisfiable or unsatisfiable. If it is valid, or unsatisfiable, provide a proof. If it is satisfiable (and not valid), then exhibit a model that interprets the concept in a non-empty set:

1. $\neg(\forall r.A \sqcup \exists r.(\neg A \sqcap \neg B))$.
2. $\exists r.(\forall s.C) \sqcap \forall r.(\exists s.\neg C)$.
3. $(\exists s.C \sqcap \exists s.D) \sqcap \forall s.(\neg C \sqcup \neg D)$.
4. $\exists s.(C \sqcap D) \sqcap (\forall s.\neg C \sqcup \forall s.\neg D)$.
5. $C \sqcap \exists r.A \sqcap \exists r.B \sqcap \neg\exists r.(A \sqcap B)$

2 Bissimulation

For each of the following pairs of models, check if they are bisimular. If yes, find the bisimulation relation, if not, find a formula that is true in the first model and false in the second.



3 $\mathcal{ALC}$ Tableaux

Check by means of tableaux whether the following subsumption is valid:

1. $\neg\forall r.A \sqcap \forall r.((\forall r.B) \sqcup A) \sqsubseteq \forall r.\neg(\exists r.A) \sqcup \exists r.(\exists r.B)$

4 $\mathcal{ALC}$ Tableaux

Which of the following statements are true? Explain your answer using tableaux.

1. $\forall r.\,(A \sqcap B) \sqsubseteq \forall r.A \sqcap \forall r.B$
2. $\forall r.A \sqcap \forall r.B \sqsubseteq \forall r.\,(A \sqcap B)$
3. $\forall r.A \sqcup \forall r.B \sqsubseteq \forall r.\,(A \sqcup B)$
4. $\forall r.\,(A \sqcup B) \sqsubseteq \forall r.A \sqcup \forall r.B$
5. $\exists r.\,(A \sqcap B) \sqsubseteq \exists r.A \sqcap \forall r.B$
6. $\exists r.\,(A \sqcup B) \sqsubseteq \exists r.A \sqcup \forall r.B$
7. $\exists r.A \sqcup \forall r.B \sqsubseteq \exists r.\,(A \sqcup B)$
8. $\exists r.A \sqcap \forall r.B \sqsubseteq \exists r.\,(A \sqcap B)$

5 $\mathcal{ALC}$ Tableaux with cyclic TBoxes

Check by means of tableaux whether the following subsumption is valid w.r.t. the TBox $\{A \sqsubseteq \exists r.(A \sqcup B)\}$:

1. $A \sqsubseteq \exists r.B$