Knowledge Representation and Reasoning

Exercises on Defaults

1 Closed World Assumption

Consider the following knowledge base:

$$\begin{aligned} KB = \{&NorthOf(york, edinburgh), \\ &NorthOf(london, nottingham), \\ &NorthOf(york, durham), \\ &NorthOf(london, york), \\ &\forall x \forall y \forall z\, (NorthOf(x, y) \wedge NorthOf(y, z) \supset NorthOf(x, z))\} \end{aligned}$$

Determine whether each of the following consequences holds. Justify your answer.

1. $KB \models_C NorthOf(london, edinburgh)$
2. $KB \models_C \neg NorthOf(london, edinburgh)$
3. $KB \models_C NorthOf(nottingham, edinburgh)$
4. $KB \models_C \neg NorthOf(nottingham, edinburgh)$

Answer:

1. $KB \models_C NorthOf(london, edinburgh)$, since $NorthOf(london, york)$, $NorthOf(york, edinburgh)$, and $\forall x \forall y \forall z\, (NorthOf(x, y) \wedge NorthOf(y, z) \supset NorthOf(x, z))$ classically entail $NorthOf(london, edinburgh)$.
2. $KB \not\models_C \neg NorthOf(london, edinburgh)$, because $KB \models_C NorthOf(london, edinburgh)$ and thus $\neg NorthOf(london, edinburgh) \notin Negs$.
3. $KB \not\models_C NorthOf(nottingham, edinburgh)$ because $NorthOf(nottingham, edinburgh)$ does not follow classically from KB (a consistent interpretation for KB which does not satisfy $NorthOf(nottingham, edinburgh)$ can be obtained by, using as domain the cities mentioned explicitly, interpreting each city as itself, and include in $NorthOf^I$ precisely the pairs of cities explicitly mentioned in KB); so its negation is in $Negs$, and since $KB \cup Negs$ is consistent, $NorthOf(nottingham, edinburgh)$ is not entailed by it.
4. $KB \models_C \neg NorthOf(nottingham, edinburgh)$ because $NorthOf(nottingham, edinburgh)$ does not follow classically from KB, so its negation occurs in $Negs$.

2 Circumscription

Consider the following knowledge base:

$$
\begin{aligned}
KB = \{ & NorthOf(milan, glasgow), \\
& NorthOf(milan, london), \\
& NorthOf(milan, moscow), \\
& glasgow \neq london \\
& london \neq moscow \\
& glasgow \neq moscow \\
& \neg ColderThan(milan, glasgow) \vee \neg ColderThan(milan, london) \\
& \forall x\, (NorthOf(milan, x) \wedge \neg Ab(x) \supset ColderThan(milan, x))\}
\end{aligned}
$$

State whether the following sentences are minimally entailed by KB. Justify your choice.

1. $ColderThan(milan, moscow)$
2. $ColderThan(milan, glasgow) \vee ColderThan(milan, london)$

Answer:

1. The minimal models of KB are those where the extension $I(Ab)$ of Ab is either $\{I(glasgow)\}$ or $\{I(london)\}$, but not both. In all of them $(I(milan), I(moscow))$ is in $I(NorthOf)$ and $I(moscow)$ is not in $I(Ab)$, so $(I(milan), I(moscow))$ is in $I(ColderThan)$.
2. The minimal models of KB are those where the extension $I(Ab)$ of Ab is either $\{I(glasgow)\}$ or $\{I(london)\}$, but not both. (Thus, note that consequently $KB \not\models_{\leq} ColderThan(milan, glasgow)$ and $KB \not\models_{\leq} ColderThan(milan, london)$.) In the former case, $(I(milan), I(london))$ is in $I(ColderThan)$, in the latter $(I(milan), I(glasgow))$ is in $I(ColderThan)$. We conclude that $KB \models_{\leq} ColderThan(milan, glasgow) \vee ColderThan(milan, london)$.

3 Default Extensions

Compute the default extensions of each of the following theories $\Delta = (W, D)$:

1. $W = \{a\}$ and $D = \left\{ \frac{a:\neg b}{c}, \frac{:\neg c}{d}, \frac{:\neg d}{e} \right\}$.
2. $W = \{a \supset c,\ b \supset c\}$ and $D = \left\{ \frac{:\neg b}{a}, \frac{:\neg a}{b}, \frac{:\neg d}{e} \right\}$.
3. $W = \{\}$ and $D = \left\{ \frac{:\neg b}{a}, \frac{:\neg a}{b}, \frac{:\neg d}{d} \right\}$.
4. $W = \{\}$ and $D = \left\{ \frac{:\neg b}{a}, \frac{:\neg a}{b}, \frac{a:\neg d}{d} \right\}$.
5. $W = \{\}$ and $D = \left\{ \frac{:\neg b}{a}, \frac{:\neg a}{b}, \frac{:\neg d}{d}, \frac{:\neg a}{d} \right\}$.
6. $W = \{p \wedge c\}$ and $D = \left\{ \frac{b\ :\ a}{a}, \frac{:\neg a}{\neg a}, \frac{:\neg a}{\neg c}, \frac{:\neg q}{b}, \frac{:\neg p}{q} \right\}$.

Answer:

1. $E = Cn(\{a, c, e\})$
2. $E_1 = Cn(\{a \supset c,\ b \supset c, a, e\})$ and $E_2 = Cn(\{a \supset c,\ b \supset c, b, e\})$
3. none
4. $E = Cn(\{b\})$
5. $E = Cn(\{b, d\})$
6. $E_1 = Cn(\{p \wedge c, a, b\})$

4 Default Extensions

> *"John wants to make a boat trip with two friends: Peter and Mary. John's mother agrees with this as long as there is a guarantee that it is safe to travel by boat with at least one of John's companions. Moreover, John's mother finds it normal that to travel by boat is safe with the company of someone who has had a course in navigation.*
>
> *Meanwhile, John's father takes cognizance of the matter and, in that regard, tells John's mother he is sure at least one of those of John's friends has had a course in navigation, though he cannot recall who".*

This knowledge may be represented by default theory $\Delta = (W, D)$

where $W = \{companion(mary), companion(peter)\}$ and D is made up of the default rules:

$$\frac{companion(X) \wedge safeWith(X) :}{agrees} \qquad \frac{course(X) : safeWith(X)}{safeWith(X)}$$

$$\frac{: \neg course(mary)}{course(peter)} \qquad \frac{: \neg course(peter)}{course(mary)}$$

1. Show that from this theory one can conclude that John's mother agrees with him making the trip, i.e. that *agrees* belongs to all extensions of Δ.
2. Suppose now that John's mother hears, from a secure source, that Peter is not trustworthy sailing boats. The incorporation of this new knowledge can be done by adding to W the fact $\neg safeWith(peter)$. Show that, after this addition, one may no longer conclude (in all extensions) that John's mother agrees with the boat trip. Also, state what can be concluded, in this new situation, about who has had a course in navigation.

Answer:
Left as an exercise.

5 Default Extensions

Consider the following situation:

> *Normally, and if not broken, the left arm is in shape. Similarly for the right arm. One of the arms (we do not know which) is broken.*

modelled by theory $\Delta = (W, D)$ where:

$$W = \{left\_broken \vee right\_broken\}$$

$$D = \left\{ \frac{:left\_good \wedge \neg left\_broken}{left\_good} \quad \frac{:right\_good \wedge \neg right\_broken}{right\_good} \right\}$$

1. Show that, contrary to one's intuitive expectations, theory Δ has an extension $E = Th(\{left\_broken \vee right\_broken, left\_good, right\_good\})$.
2. Present another formalization (in default logic) that gives the expected result, i.e. two extensions $E_1 = \{left\_broken, right\_good\}$ and $E_2 = \{right\_broken, left\_good\}$.
   Suggestion: The last sentence may be interpreted as: if one does not assume that the left arm is broken, then one must assume the right arm is; and vice-versa.

Answer:
Left as an exercise.

6 Default Extensions

Consider the following situation:

> *Whenever the sun is shining, a family normally goes to the beach on Sunday. If there is no sunshine, the family usually goes to the cinema. The son, Paul, does not like the beach much, and is happier whenever the family does not go there. The daughter, Paula, to the contrary, is happier whenever the family does not go to the cinema. Furthermore, on Sunday, the family never goes both places.*

modelled by theory $\Delta = (W, D)$ where:

$$W = \{\neg beach \vee \neg cinema\} \quad D = \left\{ \frac{:sunshine}{beach} \quad \frac{:\neg sunshine}{cinema} \quad \frac{:\neg beach}{happy(paul)} \quad \frac{:\neg cinema}{happy(paula)} \right\}$$

1. Show that, contrary to one's intuitive expectations, theory Δ has no extension. In your opinion, what is the expected result in the situation described?

2. Present another formalization (in default logic) that gives the expected result.

> **Answer:**
> Left as an exercise.

7 Default Extensions

Consider the following information:

> *Normally, both, undergraduate and graduate students study. There is someone who is a student, but we do not know whether graduate or undergraduate.*

modelled by theory $\Delta = (W, D)$ where:

$$W = \{undergraduate \vee graduate\} \quad D = \left\{ \frac{undergraduate : studies}{studies} \quad \frac{graduate : studies}{studies} \right\}$$

1. Show that, contrary to one's intuitive expectations, theory Δ does not permit to conclude that someone studies.

2. Present two alternative formalizations (in default logic) that provide the expected result, such that, in at least one of them, theory W remains as given above.
   For each of the alternatives, verify that the result is indeed as expected.

> **Answer:**
> Left as an exercise.

8 Default Extensions

Consider the following default theory $\Delta = (\{\}, D)$ where D is made up of the rules:

$$\frac{}{e} \quad \frac{e:\neg c}{d} \quad \frac{:\neg c}{\neg d} \quad \frac{:\neg a(X)}{c} \quad \frac{:\neg b(X)}{a(X)} \quad \frac{:\neg a(X)}{b(X)}$$

where X may take any natural number as value, i.e. $X \in \{0, 1, 2, \dots\}$

1. Briefly justify which are the consistent theory extensions. State whether c is true in all of them.

2. On the basis of the result obtained, comment the statement: *"In general, the computation of an extension cannot be achieved by finite approximations"*.

> **Answer:**
> Left as an exercise.