Soluções dos Exercícios Propostos

A

Capítulo 1

Noções Topológicas

1. a) $\text{int}(A) =]2,3[\cup]4,10[$

   $\text{ext}(A) =]-\infty,2[\cup]3,4[\cup]10,+\infty[$

   $\text{fr}(A) = \{2,3,4,10\}$

   $A' = [2,3] \cup [4,10]$

   $\overline{A} = [2,3] \cup [4,10]$

   Conjunto majorantes é $[10,+\infty[$

   Conjunto dos minorantes é $]-\infty,2]$

   $\sup A = 10$, não existe máximo
   $\inf(A) = 2 = \min(A)$

   b) $\text{int}(B) =]5,7[$

   $\text{ext}(B) =]-\infty,5[\cup]7,15[\cup]15,+\infty[$

   $\text{fr}(B) = \{5,7,15\}$

   $B' = [5,7]$

   $\overline{B} = [5,7] \cup \{15\}$

   Conjunto dos majorantes é $[15,+\infty[$

   Conjunto dos minorantes é $]-\infty,5]$

   $\sup B = 15 = \max(B)$
   $\inf(B) = 5$, não existe mínimo

2. a) $\text{int}(A) =]-\sqrt{50},\sqrt{50}[$

   $\text{ext}(A) =]-\infty,-\sqrt{50}[\cup]\sqrt{50},+\infty[$

   $\text{fr}(A) = \{-\sqrt{50},\sqrt{50}\}$

   $A' = [-\sqrt{50},\sqrt{50}]$

   $\overline{A} = [-\sqrt{50},\sqrt{50}]$

   b) $\text{int}(B) = \varnothing$

   $\text{ext}(B) =]-\infty,-\sqrt{50}[\cup]\sqrt{50},+\infty[$

   $\text{fr}(B) = [-\sqrt{50},\sqrt{50}]$

   $B' = [-\sqrt{50},\sqrt{50}]$

   $\overline{B} = [-\sqrt{50},\sqrt{50}]$

3. a) $\text{int}(A) = \varnothing$

   $\text{ext}(A) = \mathbb{R} \setminus (A \cup \{0,2\})$

   $\text{fr}(A) = A \cup \{0,2\}$

   $A' = \{0,2\}$

   $\overline{A} = A \cup \{0,2\}$

   b) A não é aberto nem fechado

4. $\text{int}(A \cup B) =]-\sqrt{2},2[$

   $\text{ext}(A \cup B) =]-\infty,-8[\cup]\sqrt{13},+\infty[$

   $\text{fr}(A \cup B) = [-8,-\sqrt{2}] \cup [2,\sqrt{13}]$

   $(A \cup B)' = [-8,\sqrt{13}]$

5. a) $\text{int}(D) =]\frac{1}{3},\frac{3}{4}[$

   $\text{fr}(D) = (A \setminus \{\frac{1}{2}\}) \cup B \cup \{\frac{1}{3},\frac{3}{4}\}$

   $\text{ext}(D) = \mathbb{R} \setminus (A \cup B \cup\ [\frac{1}{3},\frac{3}{4}])$

   $D' = [\frac{1}{3},\frac{3}{4}] \cup \{1,2\}$

   b) D é limitado

6. a) $\text{int}(A \cup B) =]-2,1[$

   b) $(A \cup B)' = [-2,1]$

   $\text{fr}(B) = B \cup \{-1,1\}$. É limitado

7. a) $A =]-\sqrt{2}, 0[\cup]0, \sqrt{2}[$
   b) $\sup(A \cap B) = \frac{5}{4}$, $\inf(A \cap B) = 0$

8. a) $C' = \{-1, 1, e^3\}$, $\overline{C} = \{-1, 1, e^3\} \cup C$
   b) $\text{int}(C) = \varnothing$
   c) $\text{ext}(C) = \mathbb{R} \setminus \overline{C}$

   $\sup(C) = 27$, $\inf(C) = -\frac{3}{2}$

9. a) $\text{int}(A) = \cup_{k\in\mathbb{Z}}]4k\pi, 4(k+1)\pi[$
   $\text{fr}(A) = \{4k\pi, k \in \mathbb{Z}\}$
   $\text{ext}(A) = \varnothing$
   $A' = \mathbb{R}$
   b) A é aberto

10. $\text{int}(A \cup B) =]-\frac{1}{2}, 1[$
    $\text{fr}(A \cup B) = \{-1, -\frac{\sqrt{2}}{2}, -\frac{1}{2}, 1\}$
    $\sup(A \cup B) = 1$, $\inf(A \cup B) = -1$

11. $\text{ext}(B) = \varnothing$
    $\text{fr}(B) = \{x \in \mathbb{R} : x = k\pi \vee x = (2k+1)\frac{\pi}{2}, k \in \mathbb{Z}\}$
    B é aberto

12. b) B não tem majorantes, nem supremo, nem máximo
    O conjunto dos minorantes é $]-\infty, 1]$
    $\inf(B) = 1 = \min(B)$

13. $\sup(A) = 1$, não existe $\inf(A)$

14. O conjunto dos majorantes é $[1, +\infty]$
    $\max(B) = 1$, $\inf(B) = \frac{1}{2}$

15. a) $A =]-1, 2[$
    b) $\sup(A) = 2$, $\inf(A) = -1$

16. $\text{int}(S) =]-\frac{1}{11}, \frac{1}{9}[$
    $\text{fr}(S) = \{-\frac{1}{11}, \frac{1}{9}\}$

18. a) $A =]-1, 0[\cup]0, \sqrt{2}]$
    b) $A \cup B = [-1, 0[\cup]0, \sqrt{2}]$, $A \cup B$ não é aberto nem fechado

19. a) $A = [-1, 1[$, $B =]\log 3, +\infty[$
    b) $\text{int}(B \cup C) =]\log 3, +\infty[$
    $\text{fr}(B \cup C) = \{1, \log 3\}$
    $(B \cup C)' = [\log 3, +\infty[$
    c) $B \cup C$ não é aberto nem fechado

20. a) $A = [-4, -2[\cup]-2, -1[$
    b) O limite pertence a $\overline{A}$

Sucessões

3. a) $-\dfrac{1}{5}$ b) $+\infty$ c) 1 d) 0 e) $-\dfrac{1}{4}$

4. a) 0 b) -1 c) 0 d) $\dfrac{1}{2}$ e) 1

5. c) n^n, $n!$, e^n, 2^n, n^3, $2\,n$, $\sqrt{10\,n}$, $\log(n)$

6. $\dfrac{1}{n^n}, \dfrac{1}{n!}, \dfrac{1}{e^n}, \dfrac{1}{2^n}, \dfrac{1}{n^3}, \dfrac{1}{2\,n}, \dfrac{1}{\sqrt{10\,n}}, \dfrac{1}{\log(n)}$

7. a) e^4 b) 0 c) $+\infty$ d) 1

8. a) $\dfrac{1}{2e}$ b) $\dfrac{4}{e}$

9. $p = \dfrac{1}{3e}$

10. a) 1 se $x = 2k\pi$, $k \in \mathbb{Z}$
    não existe limite se $x = (2k+1)\pi$, $k \in \mathbb{Z}$
    0 se $x \in \mathbb{R} \setminus \{2k\pi, (2k+1)\pi, k \in \mathbb{Z}\}$

    b) 0 c) $+\infty$ d) $\dfrac{2}{e}$ e) 2 f) 1 g) $+\infty$ h) $+\infty$ i) $\dfrac{1}{e}$ j) 0 k) 1

13. -1

16. a) Verdadeira
    b) Falsa
    c) Falsa

17. a) $\overline{\lim}\, u_n = +\infty$; $\underline{\lim}\, u_n = 0$

b) $\overline{\lim}\, u_n = 1$; $\underline{\lim}\, u_n = -1$

c) $\overline{\lim}\, u_n = +\infty$; $\underline{\lim}\, u_n = -1$

d) $\overline{\lim}\, u_n = 1$; $\underline{\lim}\, u_n = -1$

e) $\overline{\lim}\, u_n = +\infty$; $\underline{\lim}\, u_n = 0$

f) $\overline{\lim}\, u_n = 1$; $\underline{\lim}\, u_n = 0$

g) $\overline{\lim}\, u_n = +\infty$; $\underline{\lim}\, u_n = -\infty$

h) Se $a \in \left[\frac{\pi}{4} + 2k\pi, \frac{3\pi}{4} + 2k\pi\right]$, $k \in \mathbb{Z}$,
$\overline{\lim}\, u_n = \operatorname{sen}(a)$; $\underline{\lim}\, u_n = -\operatorname{sen}(a)$
Se $a \in \left[\frac{5\pi}{4} + 2k\pi, \frac{7\pi}{4} + 2k\pi\right]$, $k \in \mathbb{Z}$,
$\overline{\lim}\, u_n = -\operatorname{sen}(a)$; $\underline{\lim}\, u_n = \operatorname{sen}(a)$
Se $a \in \left[-\frac{\pi}{4} + 2k\pi, \frac{\pi}{4} + 2k\pi\right]$, $k \in \mathbb{Z}$,
$\overline{\lim}\, u_n = \cos(a)$; $\underline{\lim}\, u_n = -\cos(a)$
Se $a \in \left[\frac{3\pi}{4} + 2k\pi, \frac{5\pi}{4} + 2k\pi\right]$, $k \in \mathbb{Z}$,
$\overline{\lim}\, u_n = -\cos(a)$; $\underline{\lim}\, u_n = \cos(a)$

i) $\overline{\lim}\, u_n = +\infty$; $\underline{\lim}\, u_n = 0$

j) $\overline{\lim}\, u_n = +\infty$; $\underline{\lim}\, u_n = -\infty$