Cap. 5 Complementares

6)

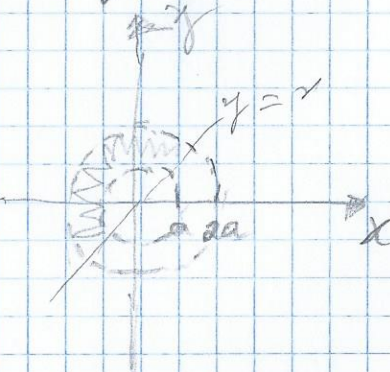
Calcule $\displaystyle\iint_D \frac{1}{\sqrt{a^2+x^2+y^2}}\,dA$ — D é interseção da coroa circular $a \le \sqrt{x^2+y^2} \le 2a,\ a>0$ com o semiplano $x-y \le 0$

$x^2+y^2=a^2 \rightarrow$

$x^2+y^2=4a^2 \rightarrow$ circunf. $C(0,0)$ e $R_1 = a$, $R_2 = 2a$

$a-2a \le 0$

$-a \le 0$ verd.



$\begin{cases} x = R\cos\theta \\ y = R\,\mathrm{sen}\,\theta \end{cases}$ $\quad \frac{\pi}{4} \le \theta \le \frac{5\pi}{4} \quad a \le R \le 2a$ $\quad |J| = R$

$$\iint_D \frac{1}{\sqrt{a^2+x^2+y^2}}\,dA = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\int_a^{2a} \frac{1}{\sqrt{a^2+R^2}}\cdot\underbrace{R}_{|J|}\,dR\,d\theta =$$

$$= \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}}\int_a^{2a} R\,(a^2+R^2)^{-\frac{1}{2}}\,dR\,d\theta = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}}\left[\frac{1}{2}\,\frac{(a^2+R^2)^{1/2}}{1/2}\right]_a^{2a} d\theta =$$

$\boxed{P_n'\ u^n = \frac{u^{n+1}}{n+1}}$

$n = -\frac{1}{2}$

$u = a^2 + a^2$

$u' = 2R$

$$= \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \left(\sqrt{a^2+4a^2} - \sqrt{a^2+a^2}\right) d\theta =$$

$$= \left(\sqrt{5}a - \sqrt{2}a\right)\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} d\theta = a\left(\sqrt{5}-\sqrt{2}\right)\pi$$

$\frac{5\pi}{4} - \frac{\pi}{4}$