

Determina

$$\begin{vmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \end{vmatrix} = D^2 f$$

$P(x,y)$

Depois de calcular as derivadas parciais, trace-se pontos e verifique condições.

Prova-se

$$\begin{vmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \end{vmatrix}$$

Condições

$$\begin{cases} |H_f| > 0 & \frac{\partial^2 f}{\partial x^2} > 0 \Rightarrow \text{mínimo} \\ |H_f| > 0 & \frac{\partial^2 f}{\partial x^2} < 0 \Rightarrow \text{máximo} \\ |H_f| < 0 & \Rightarrow \text{não há extremo} \end{cases}$$



$$\begin{aligned} x^2 + y^2 + z^2 &= 1 \\ x^2 + y^2 + z^2 - 1 &= 0 \end{aligned}$$

2

$$f(x,y,z) = x - 2y + 2z$$

$$L(x,y,z,\lambda) = f(x,y,z) + \lambda (x^2 + y^2 + z^2 - 1)$$

$$L(x,y,z,\lambda) = x - 2y + 2z + \lambda (x^2 + y^2 + z^2 - 1)$$

$$\begin{cases} \frac{\partial L}{\partial x} = 0 \\ \frac{\partial L}{\partial y} = 0 \\ \frac{\partial L}{\partial z} = 0 \\ \frac{\partial L}{\partial \lambda} = 0 \end{cases} \Leftrightarrow \begin{cases} 1 + 2x\lambda = 0 \\ -2 + 2y\lambda = 0 \\ 2 + 2z\lambda = 0 \\ x^2 + y^2 + z^2 - 1 = 0 \end{cases}$$

$$\begin{aligned} \lambda &= -\frac{1}{2x} \\ \lambda &= \frac{1}{y} \\ \lambda &= -\frac{1}{z} \end{aligned} \Rightarrow \begin{aligned} -\frac{1}{2x} &= \frac{1}{y} \Rightarrow y = -2x \\ \frac{1}{y} &= -\frac{1}{z} \Rightarrow yz = -1 \end{aligned}$$

$$\begin{aligned} x^2 + (-2x)^2 + (2x)^2 - 1 &= 0 \\ 8x^2 + 4x^2 - 1 &= 0 \end{aligned}$$

$$\begin{aligned} -2x^2 &= -2z \\ z &= x^2 \end{aligned}$$

$$t = \frac{-1 \pm \sqrt{1+32}}{16}$$

$$\begin{aligned} 8t^2 + t - 1 &= 0 \\ 8t^2 + t - 1 &= 0 \end{aligned}$$

$$\begin{aligned} x^2 &= t \\ y &= -2x \\ z &= x^2 \end{aligned}$$