

Estudar quanto à continuidade

$$\lim_{(x,y) \rightarrow (x_0,y_0)} f(x,y) = f(x_0,y_0) \quad \text{em } (x_0,y_0)$$

p.e. $\lim_{R \rightarrow 0^+} |f(R,\theta)| = 0$

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y^2 - xy}{\sqrt{x^2 + y^2}} \quad \begin{cases} x = R \cos \theta \\ y = R \sin \theta \\ R^2 = x^2 + y^2 \end{cases}$$

$$\lim_{R \rightarrow 0} \frac{(R \cos \theta)^2 (R \sin \theta)^2 - (R \cos \theta)(R \sin \theta)}{\sqrt{R^2}} =$$

$$\left| \frac{(R \cos \theta)^2 (R \sin \theta)^2 - (R \cos \theta)(R \sin \theta)}{R} \right| =$$

$$\left| \frac{R^2 (\cos^2 \theta \sin^2 \theta - \cos \theta \sin \theta)}{R} \right| =$$

$$\left| R (\cos^2 \theta \sin^2 \theta - \cos \theta \sin \theta) \right| = R |\cos^2 \theta \sin^2 \theta - \cos \theta \sin \theta|$$

$$= R \rightarrow 0^+$$

Calcule as derivadas parciais

$$\frac{\partial f}{\partial x}(x_0, y_0) = \lim_{x \rightarrow x_0} \frac{f(x, y_0) - f(x_0, y_0)}{x - x_0}$$

$$\frac{\partial f}{\partial y}(x_0, y_0) = \lim_{y \rightarrow y_0} \frac{f(x_0, y) - f(x_0, y_0)}{y - y_0}$$