SUPERFÍCIES BICÚBICAS NA FORMA PARAMÉTRICA

$$Q(s,t) = a_{11} * s^3 * t^3 + a_{12} * s^3 * t^2 + a_{13} * s^3 * t + a_{14} * s^3 +$$
$$a_{21} * s^2 * t^3 + a_{22} * s^2 * t^2 + a_{23} * s^2 * t + a_{24} * s^2 +$$
$$a_{31} * s * t^3 + a_{32} * s * t^2 + a_{33} * s * t + a_{34} * s +$$
$$a_{41} * t^3 + a_{42} * t^2 + a_{43} * t + a_{44}$$

$0 \leq s \leq 1$
$0 \leq t \leq 1$

$$Q(s,t) = S \,.\, A \,.\, T^T$$

$S = [\, s^3 \quad s^2 \quad s \quad 1 \,]$ $T = [\, t^3 \quad t^2 \quad t \quad 1 \,]$

M.Próspero

FORMA de HERMITE

Curva de Hermite:

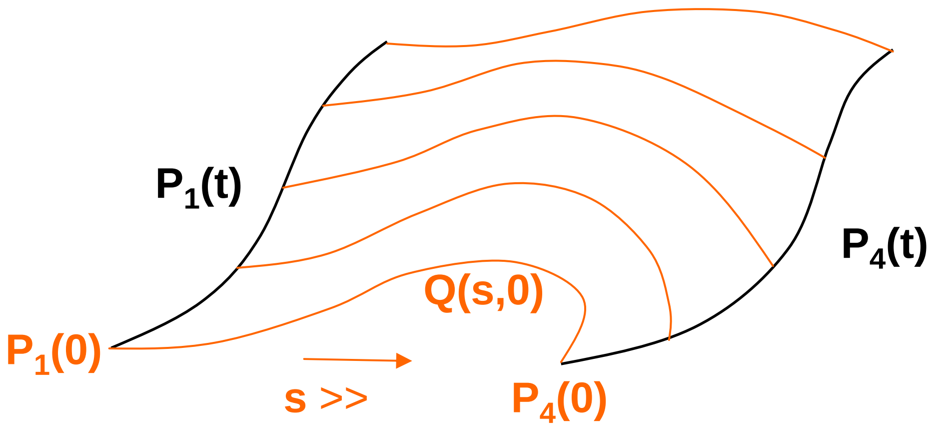
$$Q(s) = S \,.\, M_H \,.\, G_H$$

⇒

Superfície:

$$Q(s,t) = S \,.\, M_H \,.\, G_H(t) = S \,.\, M_H \,.\, \begin{bmatrix} P_1(t) \\ P_4(t) \\ R_1(t) \\ R_4(t) \end{bmatrix}$$

∴ A superfície **Q(s,t)** é interpolação entre **$P_1(t)$** e **$P_4(t)$**



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Sejam os elementos de $G_H(t)$ curvas de Hermite:

$$P_1(t) = T \,.\, M_H \,.\begin{bmatrix} g_{11} \\ g_{12} \\ g_{13} \\ g_{14} \end{bmatrix} \qquad P_4(t) = T \,.\, M_H \,.\begin{bmatrix} g_{21} \\ g_{22} \\ g_{23} \\ g_{24} \end{bmatrix}$$

$$R_1(t) = T \,.\, M_H \,.\begin{bmatrix} g_{31} \\ g_{32} \\ g_{33} \\ g_{34} \end{bmatrix} \qquad R_4(t) = T \,.\, M_H \,.\begin{bmatrix} g_{41} \\ g_{42} \\ g_{43} \\ g_{44} \end{bmatrix}$$

$$\Downarrow$$

$$\underbrace{\begin{bmatrix} P_1(t) & P_4(t) & R_1(t) & R_4(t) \end{bmatrix}}_{G_H(t)^T} = T \,.\, M_H \,.\begin{bmatrix} g_{11} & g_{21} & g_{31} & g_{41} \\ g_{12} & g_{22} & g_{32} & g_{42} \\ g_{13} & g_{23} & g_{33} & g_{43} \\ g_{14} & g_{24} & g_{34} & g_{44} \end{bmatrix}$$

Por transposição:

$$G_H(t) = \begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{22} & g_{23} & g_{24} \\ g_{31} & g_{32} & g_{33} & g_{34} \\ g_{41} & g_{42} & g_{43} & g_{44} \end{bmatrix} . M_H^T . T^T = \underline{G}_H . M_H^T . T^T$$

Por substituição em $Q(s,t) = S . M_H . G_H(t)$ **:**

$$\boxed{Q(s,t) = S . M_H . \underline{G}_H . M_H^T . T^T}$$

Superfície de Hermite

$$\underline{G}_H = \begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{22} & g_{23} & g_{24} \\ g_{31} & g_{32} & g_{33} & g_{34} \\ g_{41} & g_{42} & g_{43} & g_{44} \end{bmatrix}$$

Interpretação da <u>matriz de geometria</u> de Hermite $\underline{G}_H$:

$$\underline{G}_H = \left[\begin{array}{cc|cc} Q(0,0) & Q(0,1) & \partial Q(s,t)/\partial t \,|_{s=0,t=0} & \partial Q(s,t)/\partial t \,|_{s=0,t=1} \\ Q(1,0) & Q(1,1) & \partial Q(s,t)/\partial t \,|_{s=1,t=0} & \partial Q(s,t)/\partial t \,|_{s=1,t=1} \\ \hline \partial Q(s,t)/\partial s \,|_{s=0,t=0} & \partial Q(s,t)/\partial s \,|_{s=0,t=1} & \partial^2 Q(s,t)/\partial s\partial t \,|_{s=0,t=0} & \partial^2 Q(s,t)/\partial s\partial t \,|_{s=0,t=1} \\ \partial Q(s,t)/\partial s \,|_{s=1,t=0} & \partial Q(s,t)/\partial s \,|_{s=1,t=1} & \partial^2 Q(s,t)/\partial s\partial t \,|_{s=1,t=0} & \partial^2 Q(s,t)/\partial s\partial t \,|_{s=1,t=1} \end{array}\right]$$

As primeiras derivadas são os vectores tangente

e as segundas denominam-se *twist vectors* (vectores de torção)

Superfícies de Hermite

$$\begin{bmatrix} (-5,0,5) & (-5,0,-5) & (0,0,-10) & (0,0,-10) \\ (5,0,5) & (5,0,-5) & (0,0,-10) & (0,0,-10) \\ (10,0,0) & (10,0,0) & (0,0,0) & (0,0,0) \\ (10,0,0) & (10,0,0) & (0,0,0) & (0,0,0) \end{bmatrix}$$

(Superfície de Ferguson)

$$\begin{bmatrix} (-5,0,5) & (-5,0,-5) & (0,0,-10) & (0,0,-10) \\ (5,0,5) & (5,0,-5) & (0,0,-10) & (0,0,-10) \\ (10,0,0) & (10,0,0) & (0,200,0) & (0,200,0) \\ (10,0,0) & (10,0,0) & (0,200,0) & (0,200,0) \end{bmatrix}$$

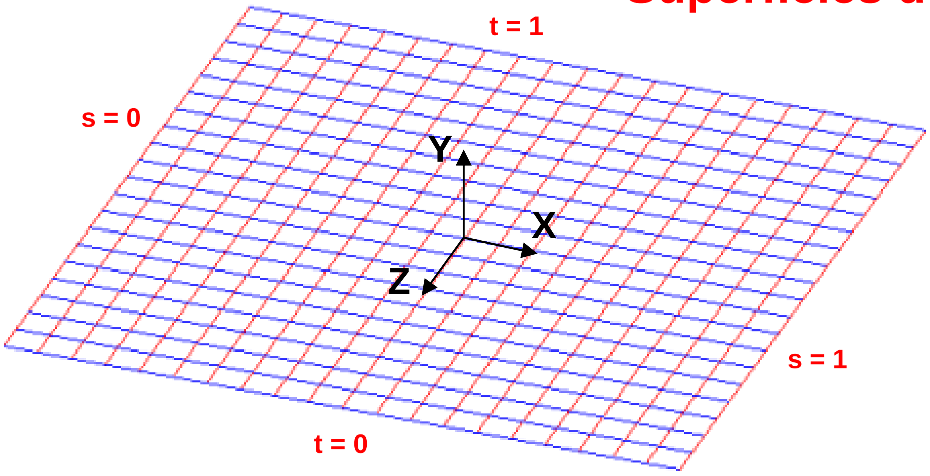
$$\begin{bmatrix} (-5,0,5) & (-5,0,-5) & (5,0,-10) & (5,0,-10) \\ (5,0,5) & (5,0,-5) & (5,0,-10) & (5,0,-10) \\ (0,10,0) & (0,10,0) & (0,0,0) & (0,0,0) \\ (0,-10,0) & (0,-10,0) & (0,0,0) & (0,0,0) \end{bmatrix}$$

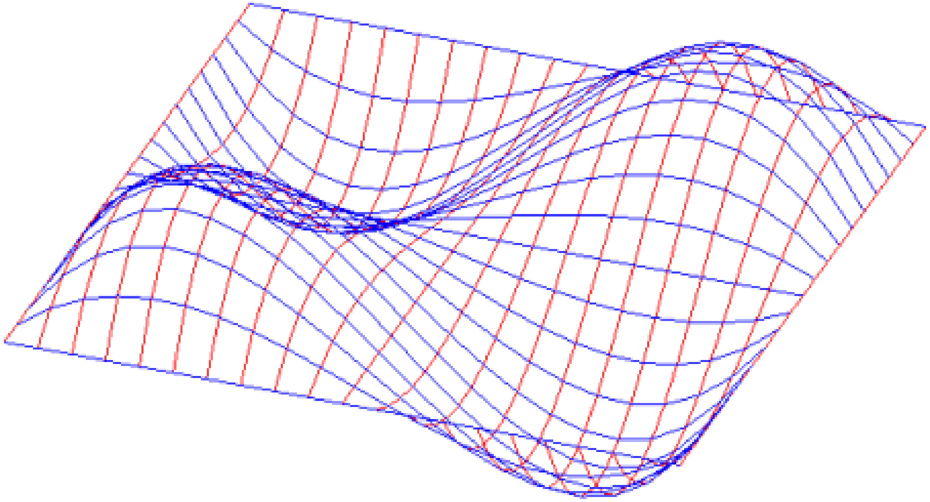
(Superfície de Ferguson)

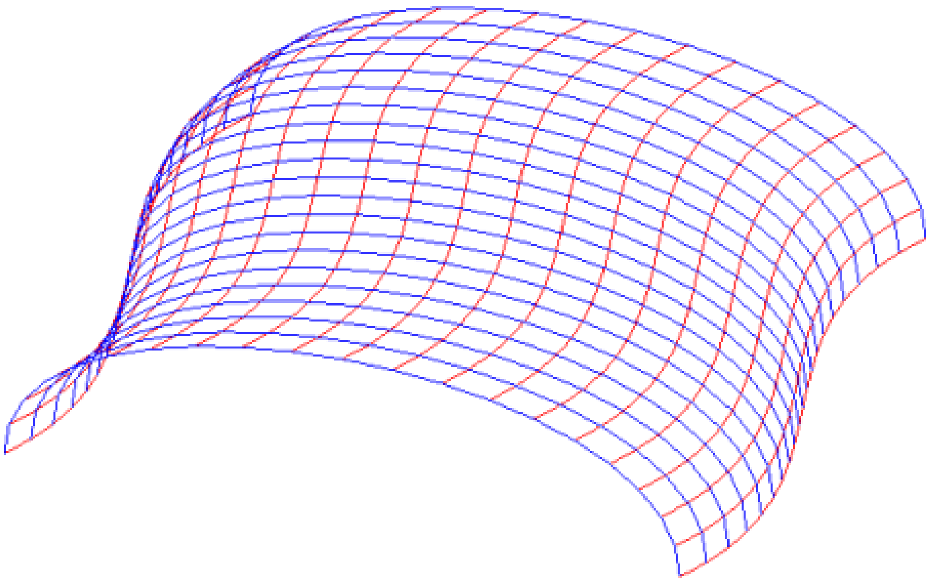
$$\begin{bmatrix} (-5,0,5) & (-5,0,-5) & (5,0,-10) & (5,0,-10) \\ (5,0,5) & (5,0,-5) & (5,0,-10) & (5,0,-10) \\ (0,10,0) & (0,10,0) & (0,200,0) & (0,200,0) \\ (0,-10,0) & (0,-10,0) & (0,200,0) & (0,200,0) \end{bmatrix}$$

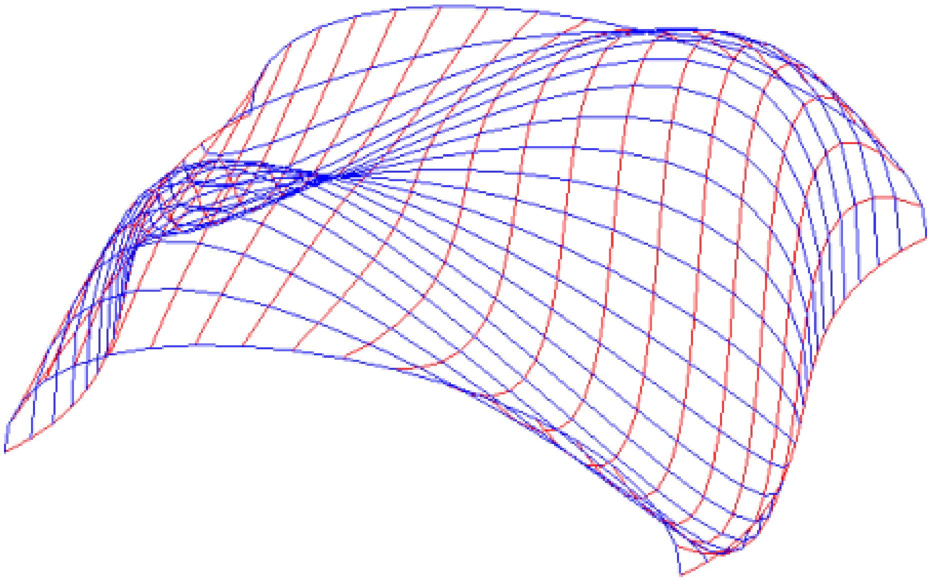
M.Próspero

Superfícies de Hermite







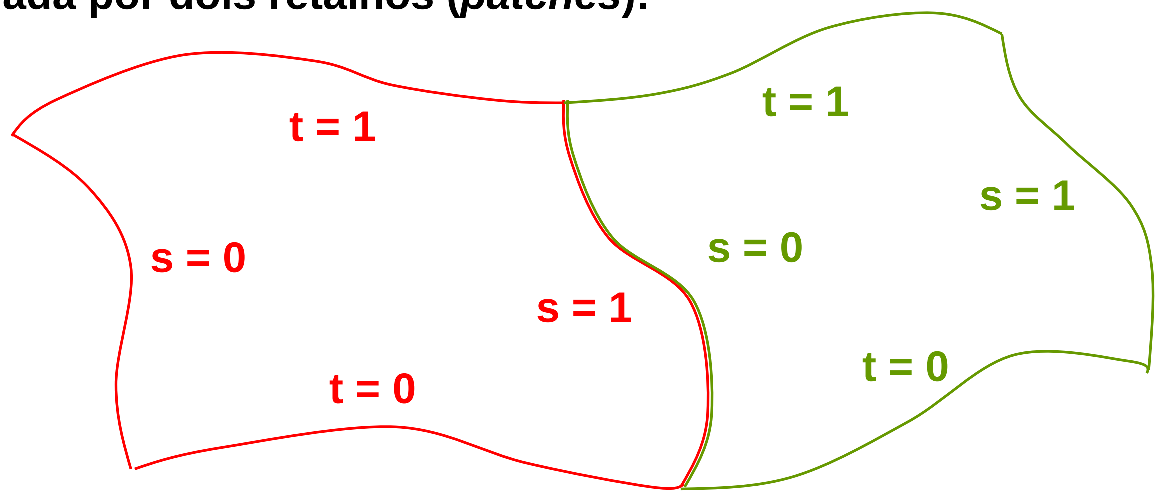


Exercício: Esboçar os vectores presentes em cada matriz e interpretá-los geometricamente.

M.Próspero

Continuidade Geométrica e Continuidade Paramétrica

Superfície formada por dois retalhos (*patches*):



Para garantir G^0 $\rightarrow$ $\underline{G}_H[2, i] = \underline{G}_H[1, i]$ $\quad i = 1..4$

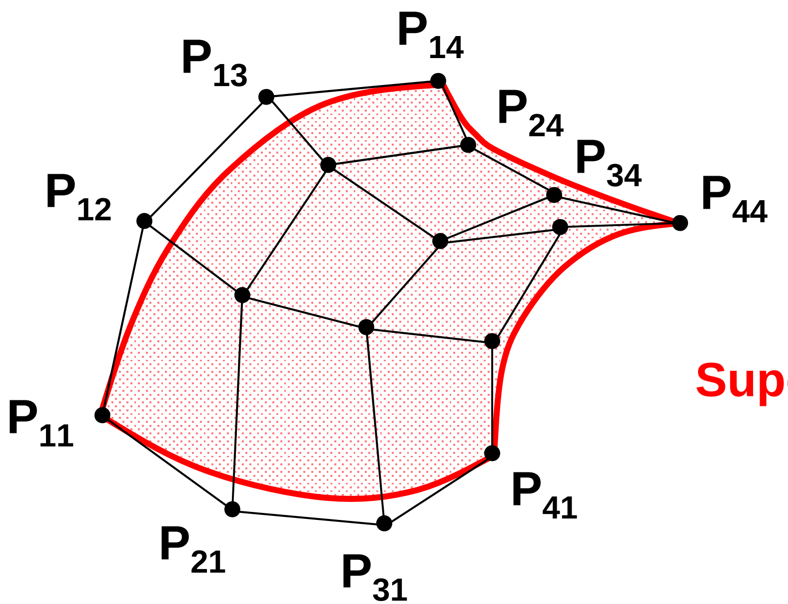
Para garantir G^1 $\rightarrow$ $\underline{G}_H[2, i] = \underline{G}_H[1, i]$ and $\underline{G}_H[4, i] = k \cdot \underline{G}_H[3, i]$ $\quad k > 0$

$k \neq 1$ $\rightarrow$ C^0

$k = 1$ $\rightarrow$ C^1

FORMA de BÉZIER

$$Q(s,t) = S \,.\, M_B \,.\, \underline{G} \,.\, M_B^{T} \,.\, T^{T}$$

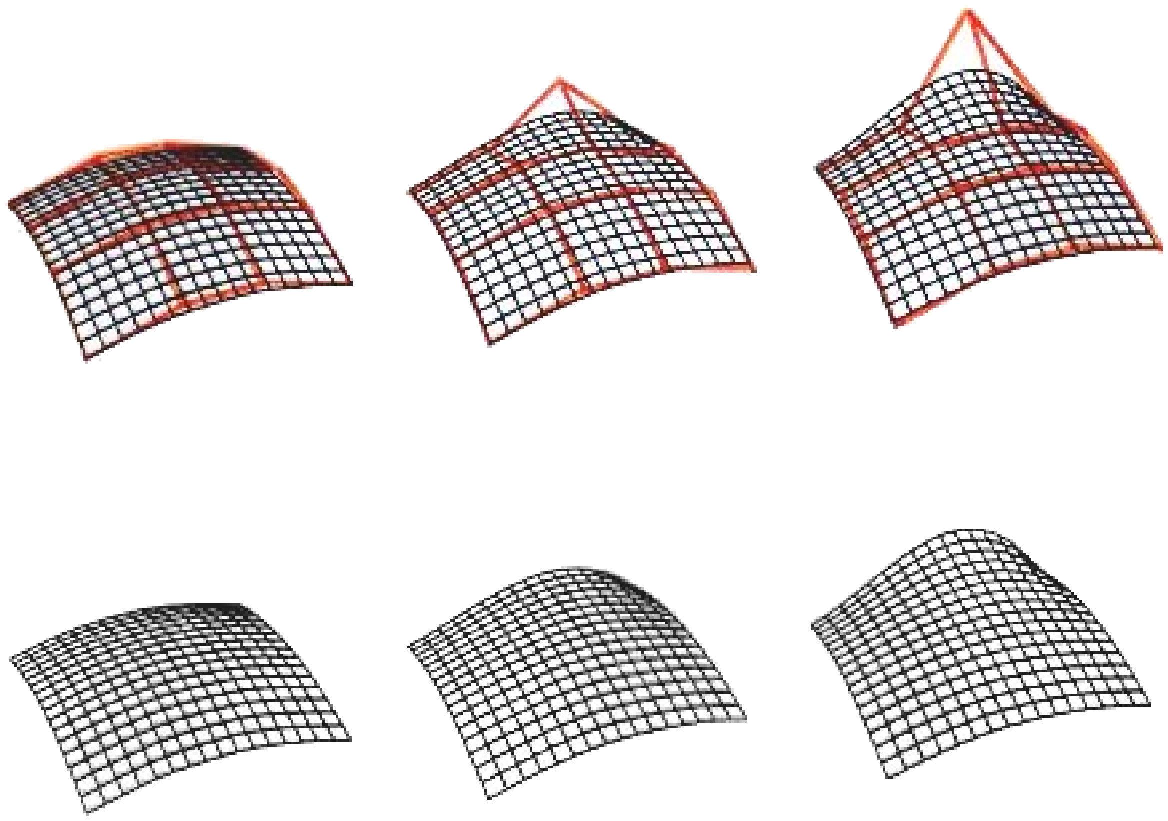


Superfície de Bézier

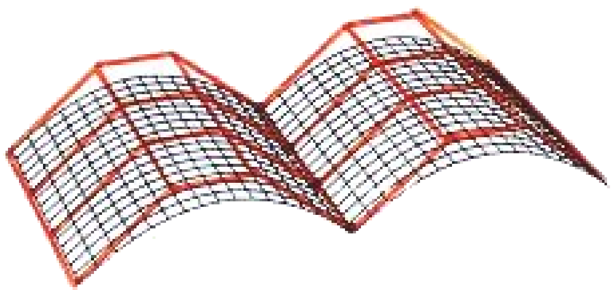
A matriz de geometria de Bézier é constituída por 16 pontos.

A superfície gerada é interior ao *convex hull* dos pontos de controlo.

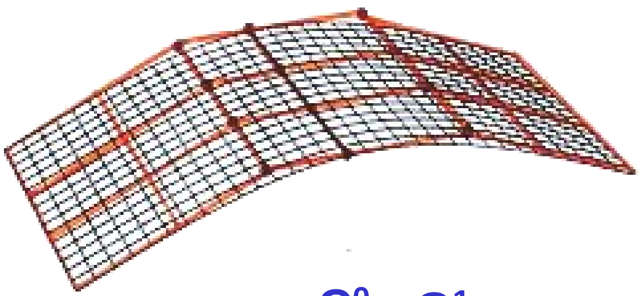
Superfícies de Bézier



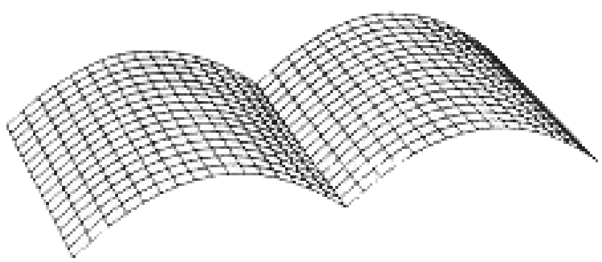
Superfícies de Bézier

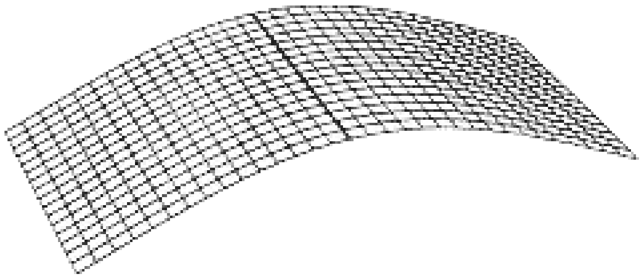


C^0 G^0



C^0 G^1





Garante-se C^0G^0 , C^0G^1 ou C^1G^1 por construção geométrica

(via coincidência de pontos, colinearidade e controlo de distâncias)

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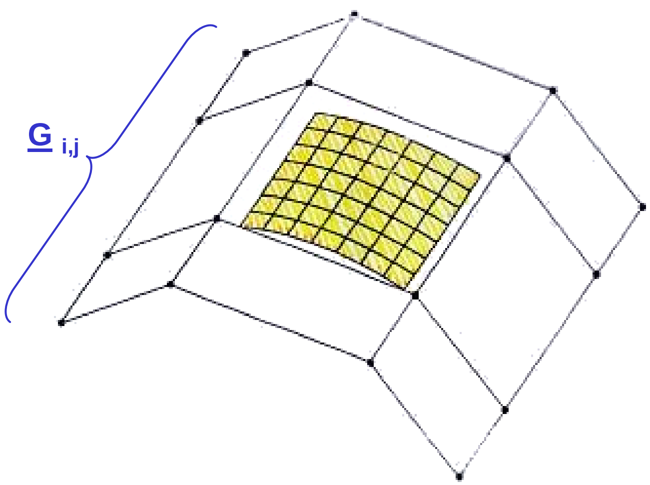
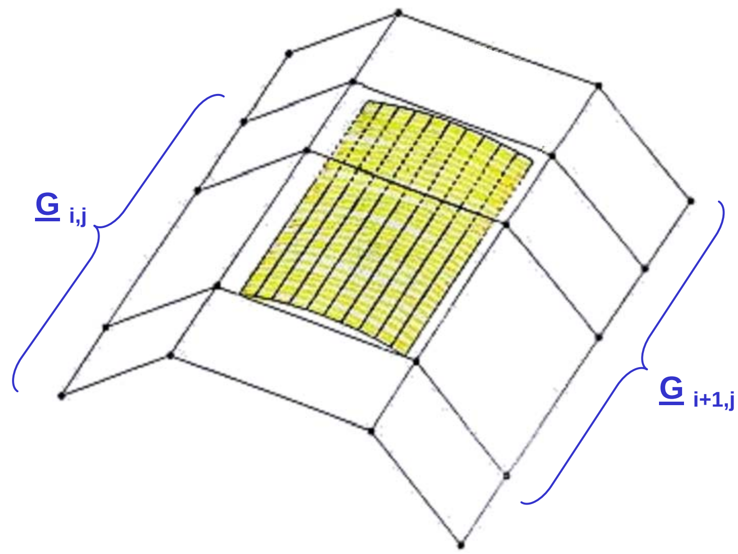
FORMA de B-SPLINE

$$Q(s,t) = S \cdot M_{Bs} \cdot \underline{G} \cdot M_{Bs}^{T} \cdot T^{T}$$

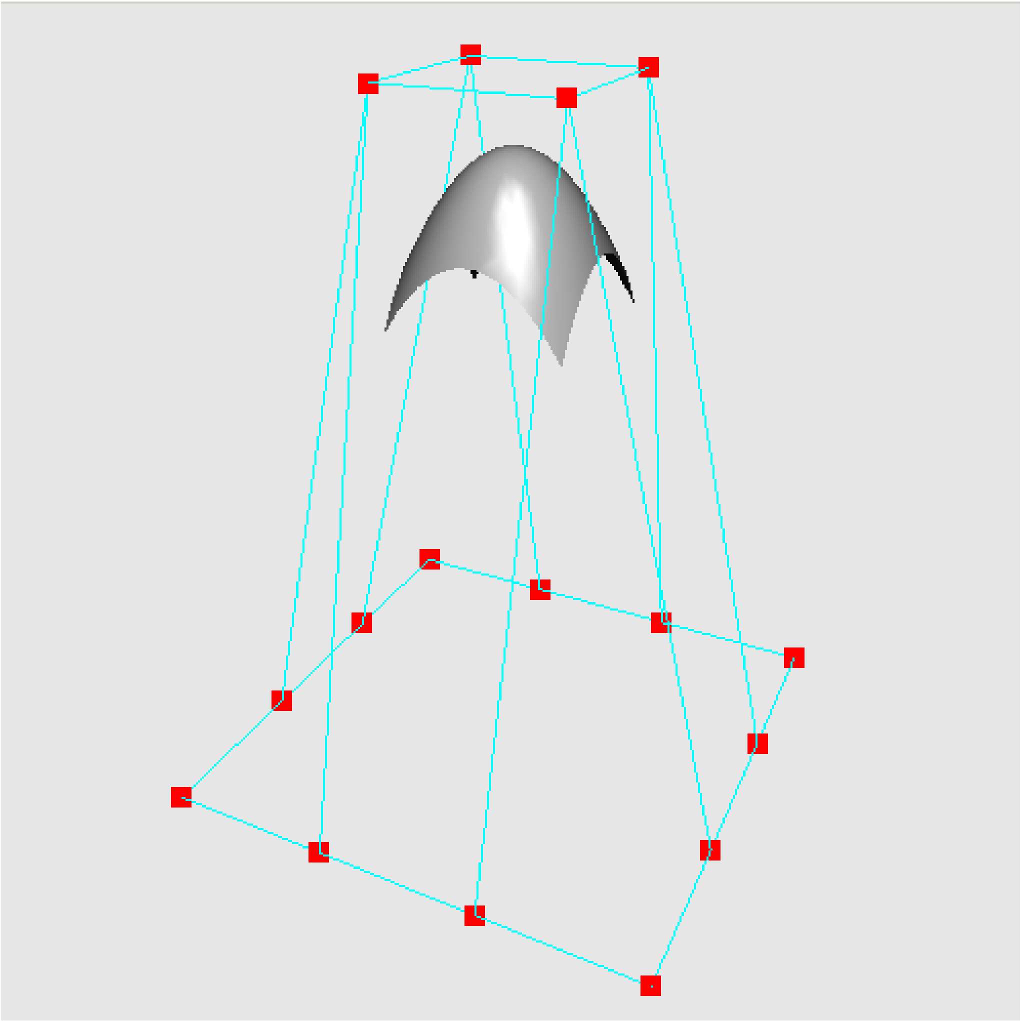




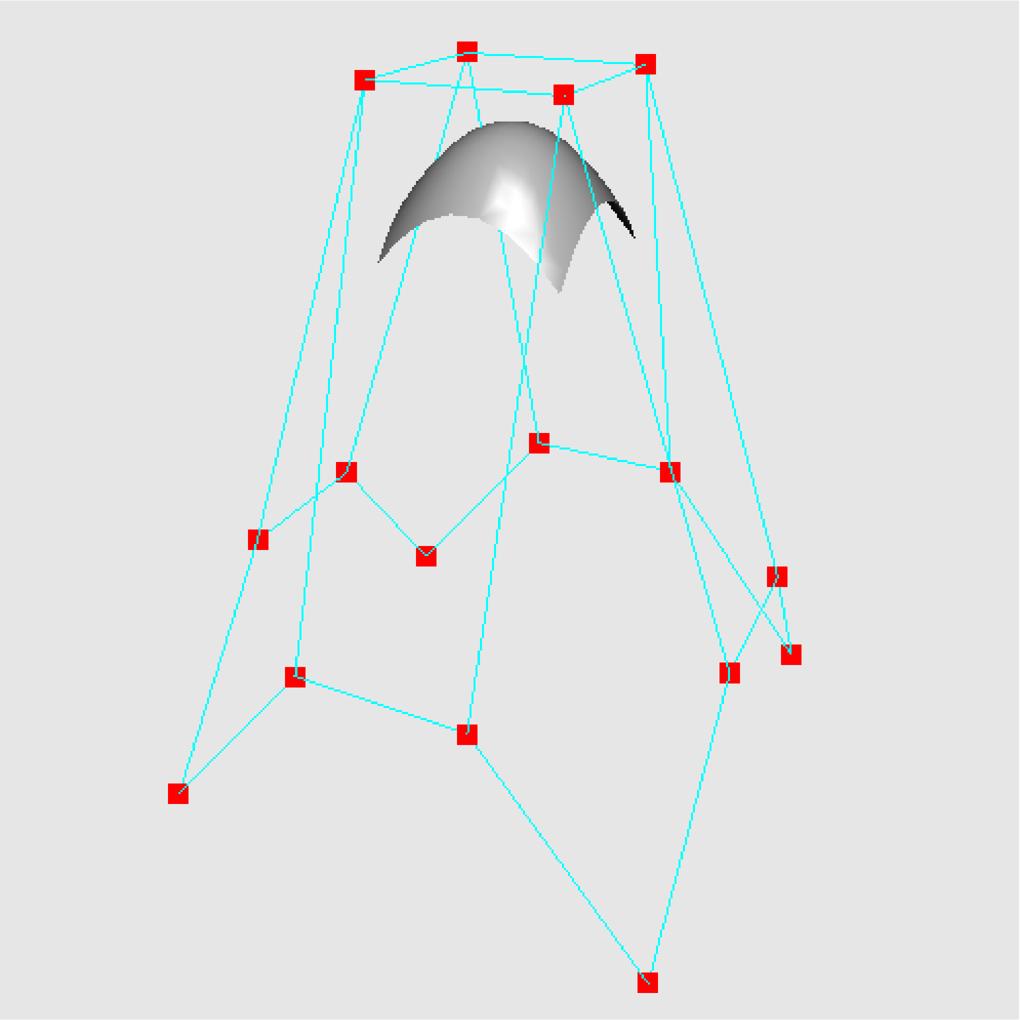
Garante-se C^2 pela utilização dos pontos de controlo via matriz de geometria $\underline{G}_{i,j}$

à semelhança das curvas B-spline com o vector de geometria G_i

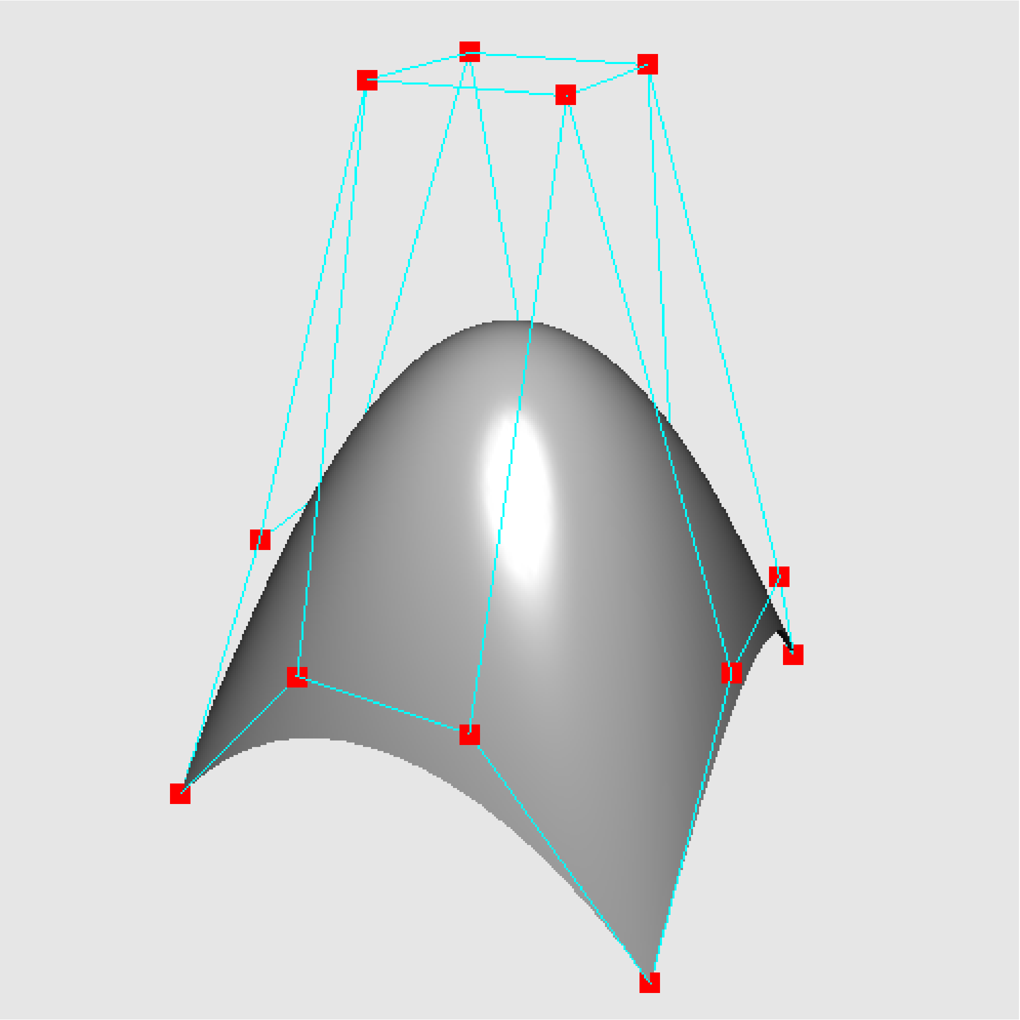
EXEMPLOS



B-spline



B-spline



Bézier

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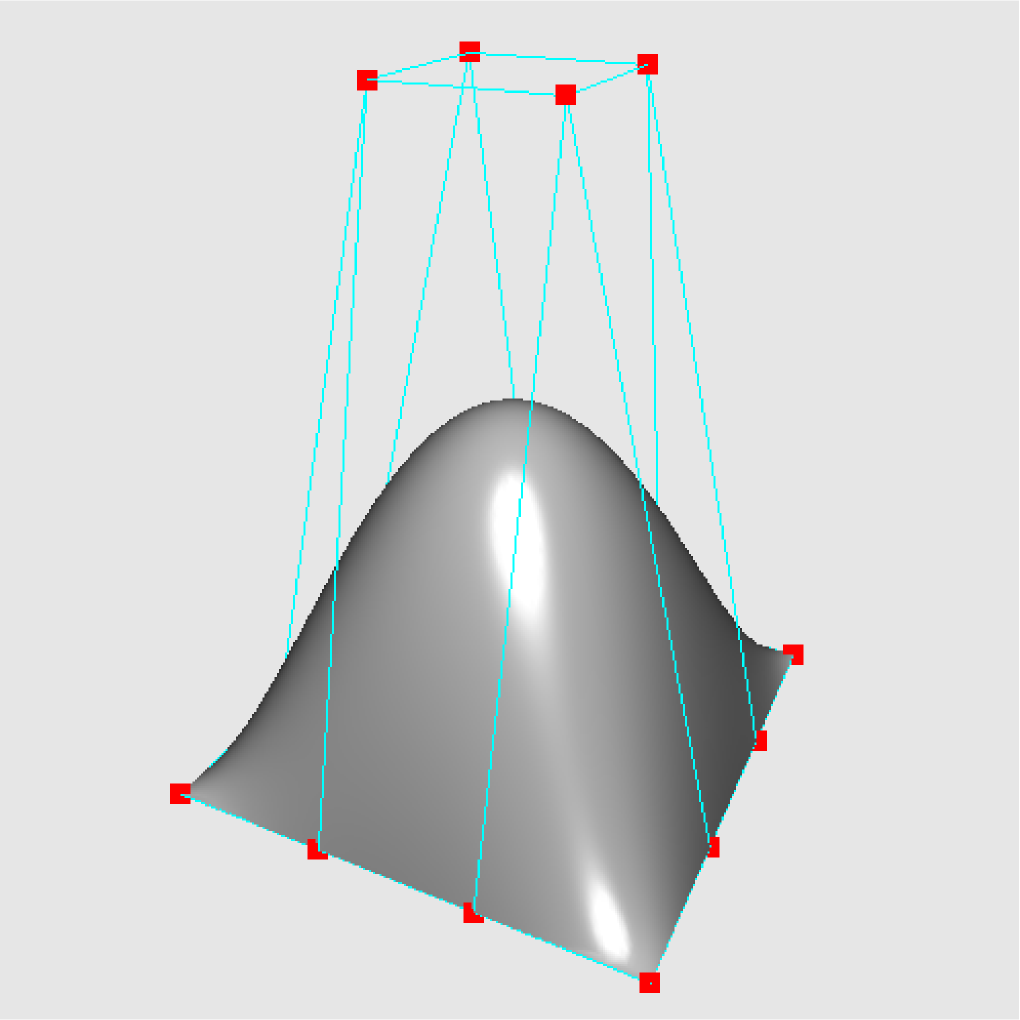

Bézier

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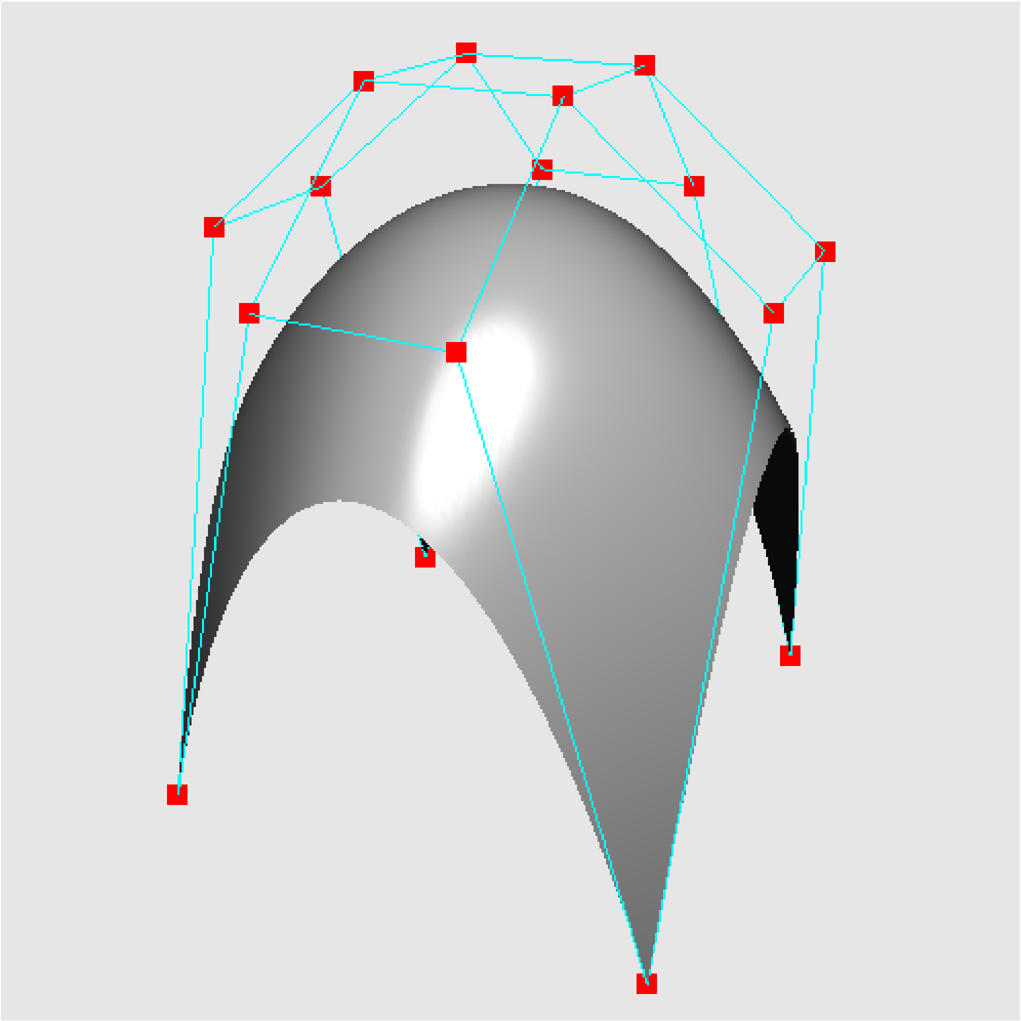
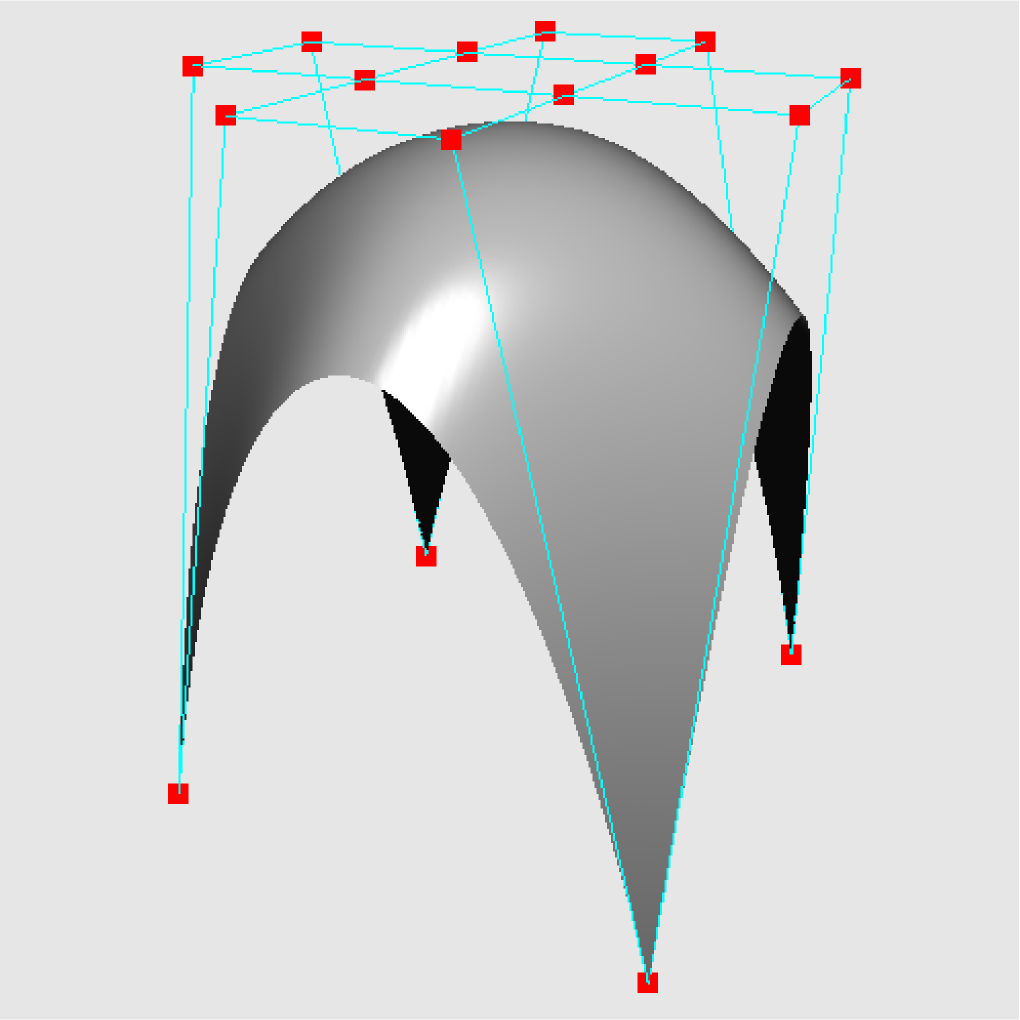


Bézier

M.Próspero

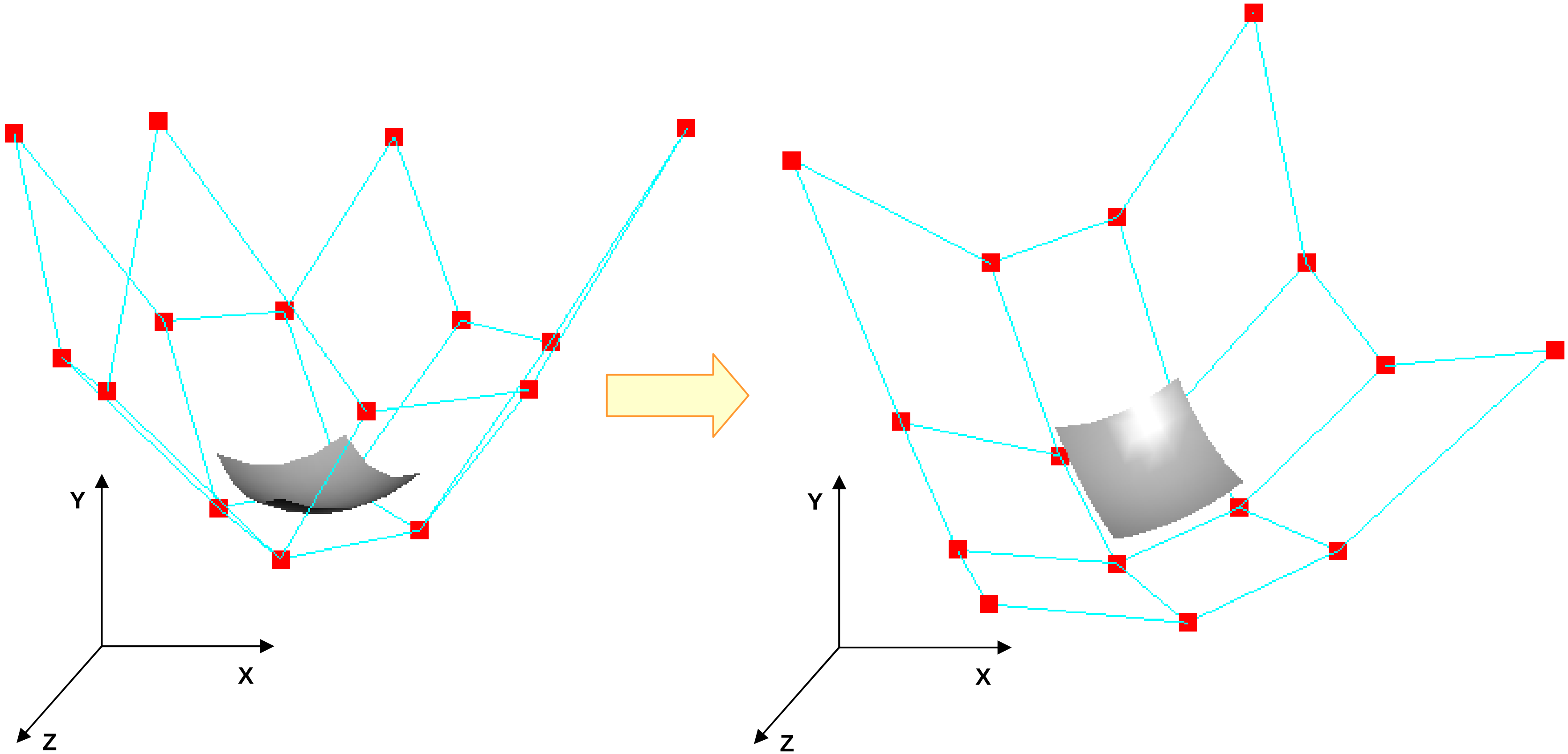


Bézier

O vector normal à superfície em cada ponto é imprescindível para a obtenção do sombreamento.

TRANSFORMAÇÕES GEOMÉTRICAS DE CURVAS E SUPERFÍCIES

S, R ou T aplicam-se aos coeficientes dos vectores de geometria ou das matrizes de geometria.



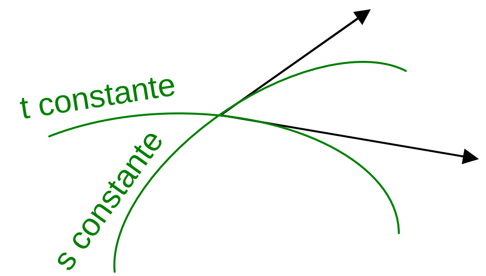
(As rotações apenas foram aplicadas aos 16 pontos de controlo e a superfície foi novamente gerada)

PLANO TANGENTE A UMA SUPERFÍCIE BICÚBICA

Vector tangente segundo s :

$$\frac{\partial}{\partial s}\,\mathbf{Q(s,t)} = \frac{\partial}{\partial s}\,(\,\mathbf{S}\,.\,\mathbf{M}\,.\,\underline{\mathbf{G}}\,.\,\mathbf{M}^{T}\,.\,\mathbf{T}^{T}\,)$$

$$= [\;3s^2\;\;2s\;\;1\;\;0\;]\,.\,\mathbf{M}\,.\,\underline{\mathbf{G}}\,.\,\mathbf{M}^{T}\,.\,\mathbf{T}^{T}$$



Vector tangente segundo t :

$$\frac{\partial}{\partial t}\,\mathbf{Q(s,t)} = \frac{\partial}{\partial t}\,(\,\mathbf{S}\,.\,\mathbf{M}\,.\,\underline{\mathbf{G}}\,.\,\mathbf{M}^{T}\,.\,\mathbf{T}^{T}\,)$$

$$= \mathbf{S}\,.\,\mathbf{M}\,.\,\underline{\mathbf{G}}\,.\,\mathbf{M}^{T}\,.\begin{bmatrix} 3t^2 \\ 2t \\ 1 \\ 0 \end{bmatrix}$$

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VECTOR NORMAL A UMA SUPERFÍCIE BICÚBICA

Vector normal :

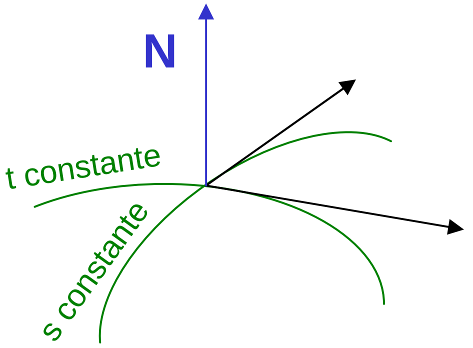
$$\frac{\partial}{\partial s} \mathbf{Q(s,t)} \times \frac{\partial}{\partial t} \mathbf{Q(s,t)} = \begin{vmatrix} \bar{\mathbf{i}} & \bar{\mathbf{j}} & \bar{\mathbf{k}} \\ x_s & y_s & z_s \\ x_t & y_t & z_t \end{vmatrix}$$

produto externo

$\therefore$

Polinómios de avaliação dispendiosa

(biquintos = 2 variáveis + grau 5)



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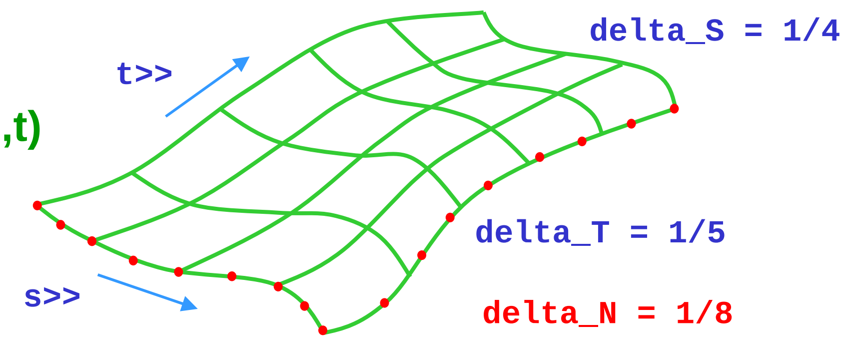
AVALIAÇÃO DE UMA SUPERFÍCIE BICÚBICA

Recorde-se que

$$\begin{aligned} Q(s,t) = S \,.\, A \,.\, T^T = \;& a_{11} * s^3 * t^3 + a_{12} * s^3 * t^2 + a_{13} * s^3 * t + a_{14} * s^3 + \\ & a_{21} * s^2 * t^3 + a_{22} * s^2 * t^2 + a_{23} * s^2 * t + a_{24} * s^2 + \\ & a_{31} * s * t^3 + a_{32} * s * t^2 + a_{33} * s * t + a_{34} * s + \\ & a_{41} * t^3 + a_{42} * t^2 + a_{43} * t + a_{44} \end{aligned}$$

retalho (*patch*) **Q(s,t)**



Algoritmo de visualização por curvas:

```
for (patch=first to last)
    for (s=0 to 1 step=delta_S)
        for (t=delta_N to 1 step=delta_N)
            Line_3D( Q(s,t-delta_N), Q(s,t) )
    for (t=0 to 1 step=delta_T)
        for (s=delta_N to 1 step=delta_N)
            Line_3D( Q(s,t-delta_N), Q(s,t) )
```

delta_N ≤ delta_T

delta_N ≤ delta_S