SUPERFÍCIES BICÚBICAS NA FORMA PARAMÉTRICA

$$Q(s,t) = a_{11} * s^3 * t^3 + a_{12} * s^3 * t^2 + a_{13} * s^3 * t + a_{14} * s^3 +$$
$$a_{21} * s^2 * t^3 + a_{22} * s^2 * t^2 + a_{23} * s^2 * t + a_{24} * s^2 +$$
$$a_{31} * s * t^3 + a_{32} * s * t^2 + a_{33} * s * t + a_{34} * s +$$
$$a_{41} * t^3 + a_{42} * t^2 + a_{43} * t + a_{44}$$

$0 \leq s \leq 1$

$0 \leq t \leq 1$

$$Q(s,t) = S \, . \, A \, . \, T^T$$

$S = [\ s^3 \quad s^2 \quad s \quad 1\ ]$

$T = [\ t^3 \quad t^2 \quad t \quad 1\ ]$

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FORMA de HERMITE

Curva de Hermite:

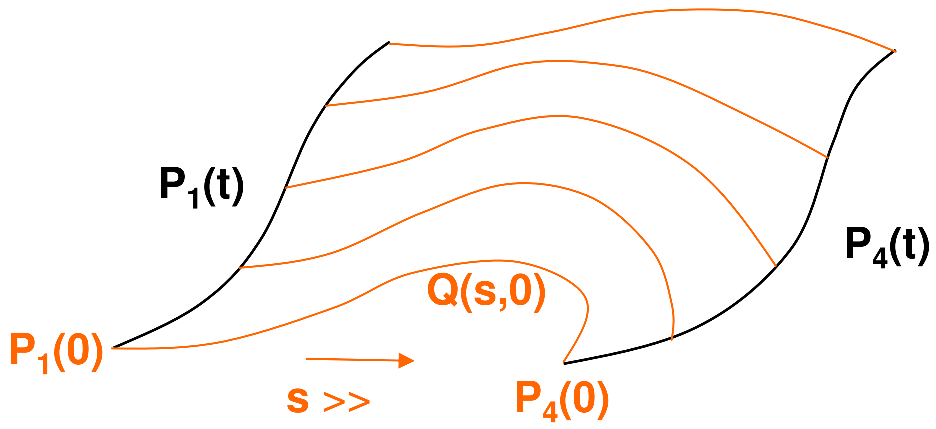
$$Q(s) = S \cdot M_H \cdot G_H$$

⇒

Superfície:

$$Q(s,t) = S \cdot M_H \cdot G_H(t) = S \cdot M_H \cdot \begin{bmatrix} P_1(t) \\ P_4(t) \\ R_1(t) \\ R_4(t) \end{bmatrix}$$

∴ A superfície **Q(s,t)** é interpolação entre **P_1(t)** e **P_4(t)**



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Sejam os elementos de $G_H(t)$ curvas de Hermite:

$$P_1(t) = T \cdot M_H \cdot \begin{bmatrix} g_{11} \\ g_{12} \\ g_{13} \\ g_{14} \end{bmatrix} \qquad P_4(t) = T \cdot M_H \cdot \begin{bmatrix} g_{21} \\ g_{22} \\ g_{23} \\ g_{24} \end{bmatrix}$$

$$R_1(t) = T \cdot M_H \cdot \begin{bmatrix} g_{31} \\ g_{32} \\ g_{33} \\ g_{34} \end{bmatrix} \qquad R_4(t) = T \cdot M_H \cdot \begin{bmatrix} g_{41} \\ g_{42} \\ g_{43} \\ g_{44} \end{bmatrix}$$

$$\begin{bmatrix} P_1(t) & P_4(t) & R_1(t) & R_4(t) \end{bmatrix} = T \cdot M_H \cdot \begin{bmatrix} g_{11} & g_{21} & g_{31} & g_{41} \\ g_{12} & g_{22} & g_{32} & g_{42} \\ g_{13} & g_{23} & g_{33} & g_{43} \\ g_{14} & g_{24} & g_{34} & g_{44} \end{bmatrix}$$

$G_H(t)^T$

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Por transposição:

$$G_H(t) = \begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{22} & g_{23} & g_{24} \\ g_{31} & g_{32} & g_{33} & g_{34} \\ g_{41} & g_{42} & g_{43} & g_{44} \end{bmatrix} . M_H^T . T^T = \underline{G}_H . M_H^T . T^T$$

Por substituição em $Q(s,t) = S . M_H . G_H(t)$ **:**

$$Q(s,t) = S . M_H . \underline{G}_H . M_H^T . T^T$$

Superfície de Hermite

$$\underline{G}_H = \begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{22} & g_{23} & g_{24} \\ g_{31} & g_{32} & g_{33} & g_{34} \\ g_{41} & g_{42} & g_{43} & g_{44} \end{bmatrix}$$

Interpretação da <u>matriz de geometria</u> de Hermite $\underline{G}_H$:

$$\underline{G}_H = \left[\begin{array}{cc|cc} Q(0,0) & Q(0,1) & \partial Q(s,t)/\partial t\,|_{s=0,t=0} & \partial Q(s,t)/\partial t\,|_{s=0,t=1} \\ Q(1,0) & Q(1,1) & \partial Q(s,t)/\partial t\,|_{s=1,t=0} & \partial Q(s,t)/\partial t\,|_{s=1,t=1} \\ \hline \partial Q(s,t)/\partial s\,|_{s=0,t=0} & \partial Q(s,t)/\partial s\,|_{s=0,t=1} & \partial^2 Q(s,t)/\partial s\partial t\,|_{s=0,t=0} & \partial^2 Q(s,t)/\partial s\partial t\,|_{s=0,t=1} \\ \partial Q(s,t)/\partial s\,|_{s=1,t=0} & \partial Q(s,t)/\partial s\,|_{s=1,t=1} & \partial^2 Q(s,t)/\partial s\partial t\,|_{s=1,t=0} & \partial^2 Q(s,t)/\partial s\partial t\,|_{s=1,t=1} \end{array}\right]$$

As primeiras derivadas são os vetores tangentes

e as segundas derivadas denominam-se *twist vectors* (vetores de torção)

Superfícies de Hermite

(−5,0,5)	(−5,0,−5)	(0,0,−10)	(0,0,−10)
(5,0,5)	(5,0,−5)	(0,0,−10)	(0,0,−10)
(10,0,0)	(10,0,0)	(0,0,0)	(0,0,0)
(10,0,0)	(10,0,0)	(0,0,0)	(0,0,0)

(Superfície de Ferguson)

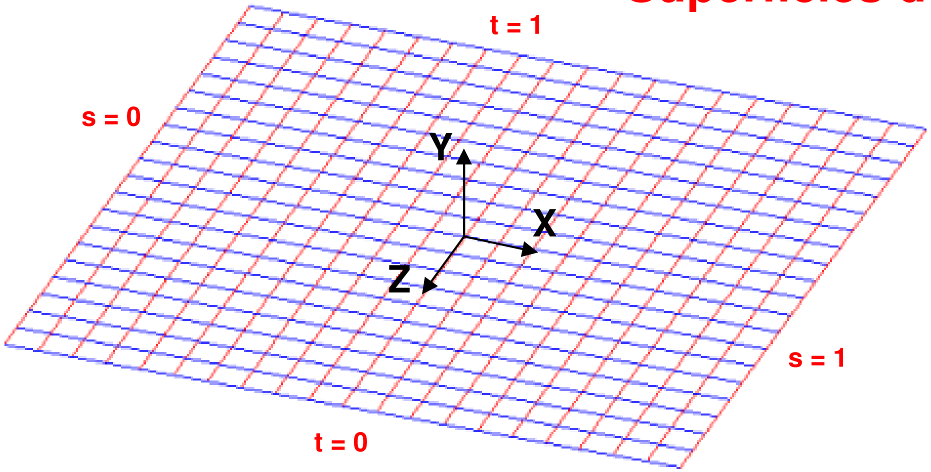
(−5,0,5)	(−5,0,−5)	(0,0,−10)	(0,0,−10)
(5,0,5)	(5,0,−5)	(0,0,−10)	(0,0,−10)
(10,0,0)	(10,0,0)	(0,200,0)	(0,200,0)
(10,0,0)	(10,0,0)	(0,200,0)	(0,200,0)

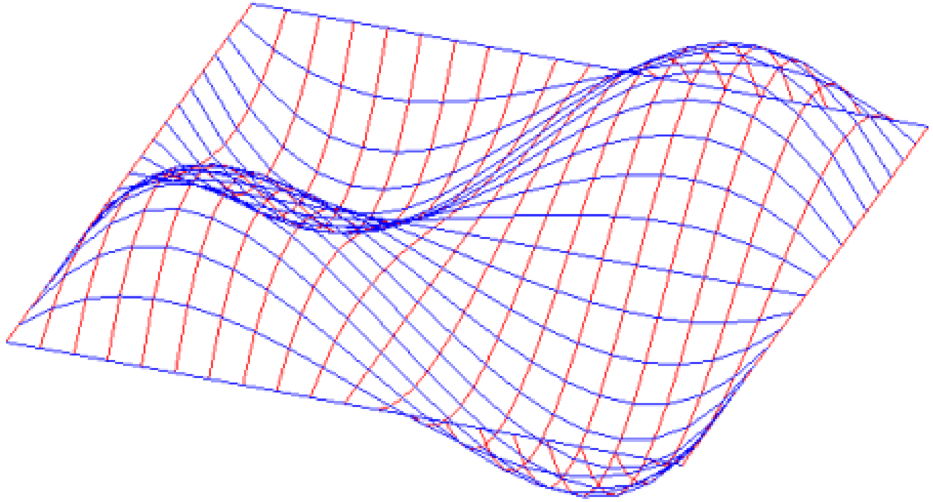
(−5,0,5)	(−5,0,−5)	(5,0,−10)	(5,0,−10)
(5,0,5)	(5,0,−5)	(5,0,−10)	(5,0,−10)
(0,10,0)	(0,10,0)	(0,0,0)	(0,0,0)
(0,−10,0)	(0,−10,0)	(0,0,0)	(0,0,0)

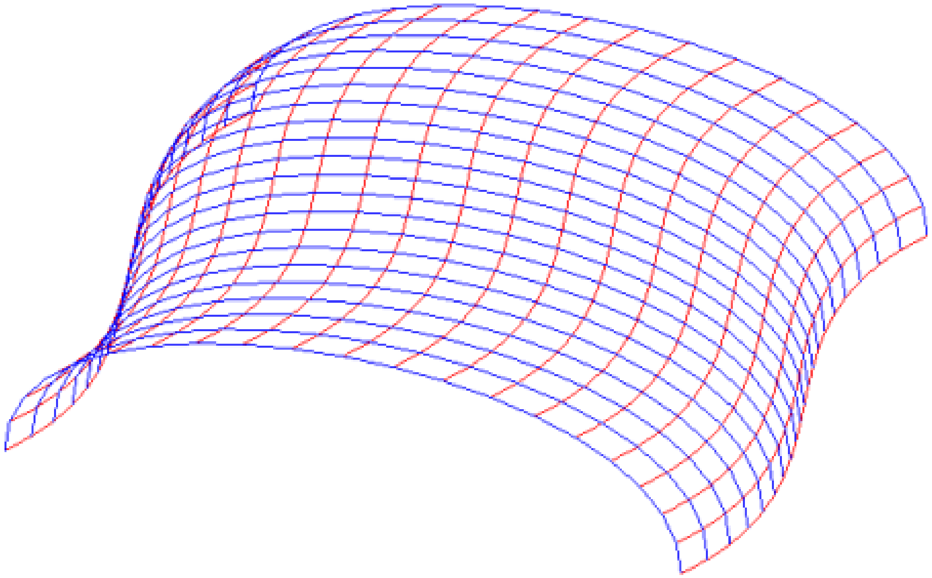
(Superfície de Ferguson)

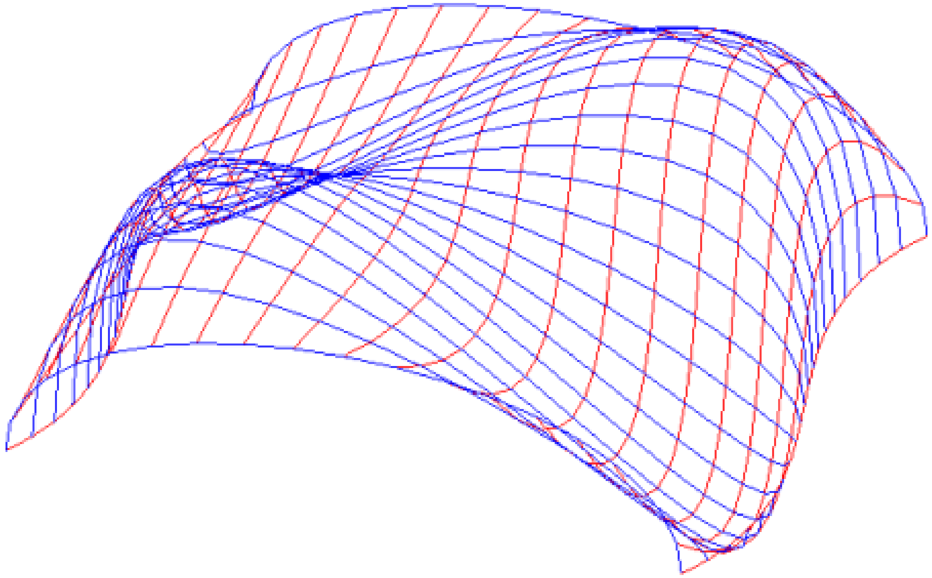
(−5,0,5)	(−5,0,−5)	(5,0,−10)	(5,0,−10)
(5,0,5)	(5,0,−5)	(5,0,−10)	(5,0,−10)
(0,10,0)	(0,10,0)	(0,200,0)	(0,200,0)
(0,−10,0)	(0,−10,0)	(0,200,0)	(0,200,0)

Superfícies de Hermite









Exercício: Esboçar os vetores presentes em cada matriz e interpretar geometricamente o resultado.

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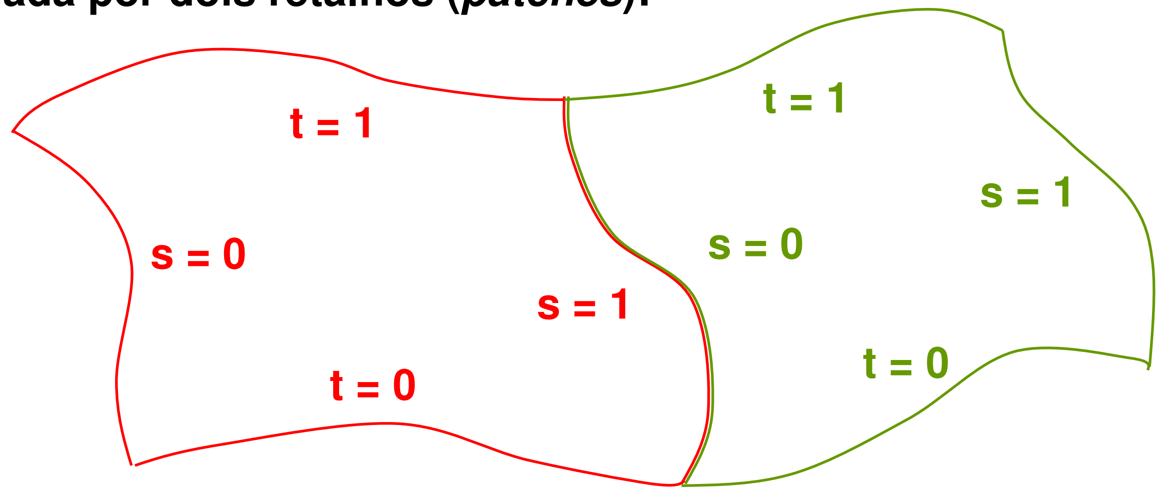
Continuidade Geométrica e Continuidade Paramétrica

Superfície formada por dois retalhos (*patches*):



Para garantir G^0

$$G_H[2, i] = G_H[1, i]$$

$i = 1..4$

Para garantir G^1

$$G_H[4, i] = k \cdot G_H[3, i]$$

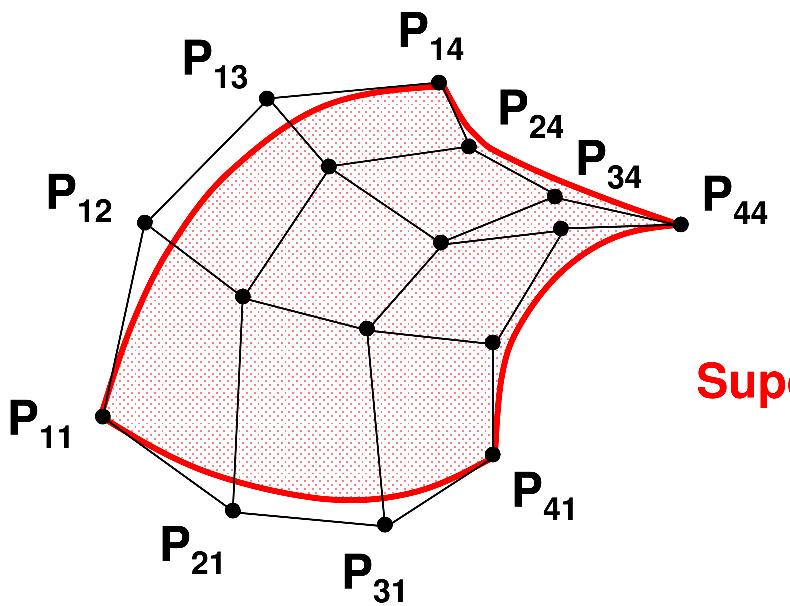
Significa que, para cada valor de t (de 0 a 1), existe **$k > 0$** tal que **$k = R_4(t) / R_1(t)$**

$k \neq 1$ → **C^0**

$k = 1$ → **C^1**

FORMA de BÉZIER

$$Q(s,t) = S \cdot M_B \cdot \underline{G} \cdot M_B^T \cdot T^T$$



Superfície de Bézier

A matriz de geometria de Bézier <u>G</u> é constituída por 16 pontos.

A superfície gerada é interior ao *convex hull* dos pontos de controlo.

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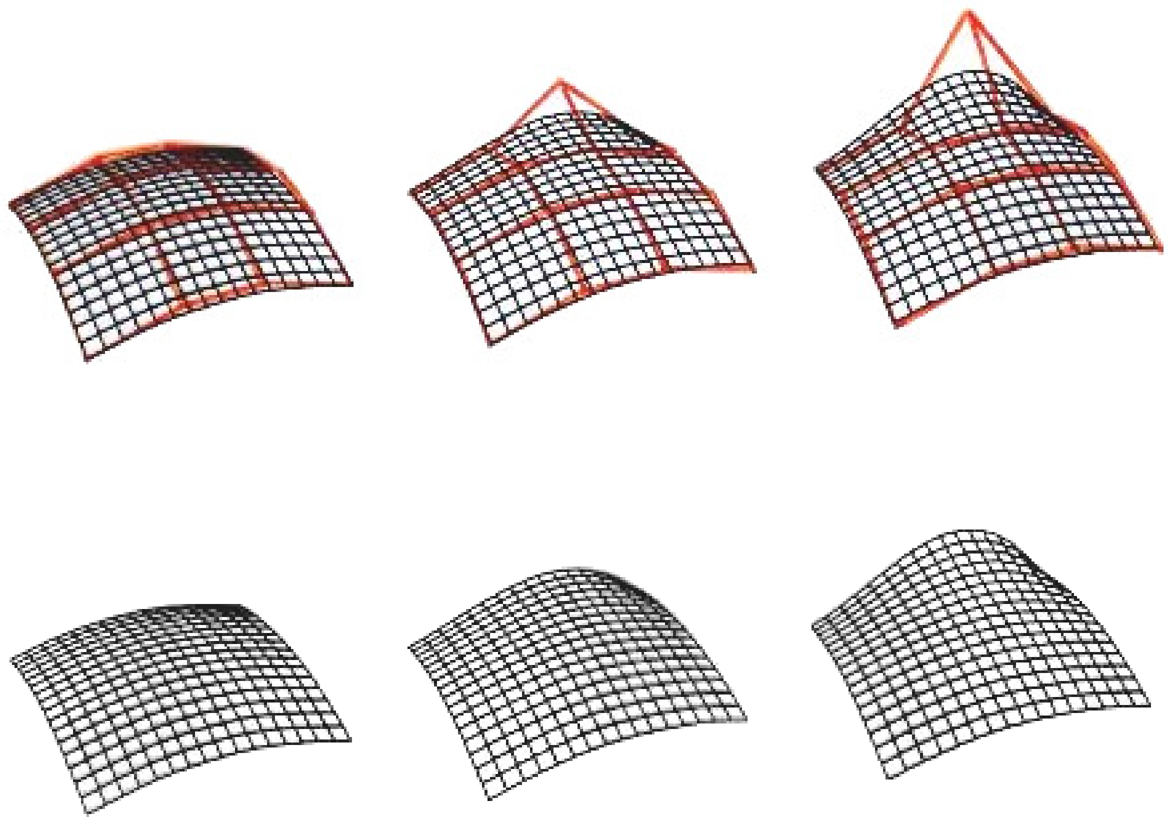
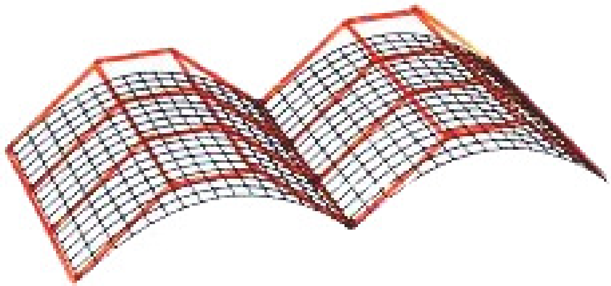
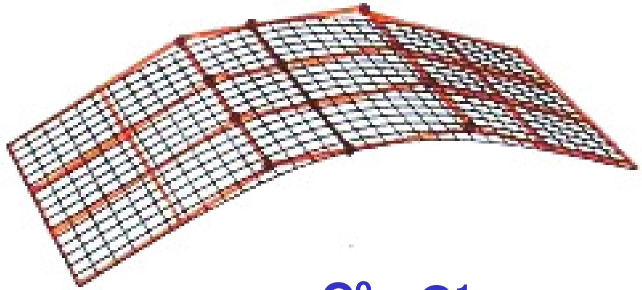
Superfícies de Bézier



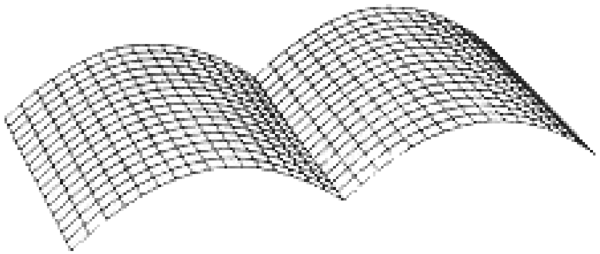
Superfícies de Bézier

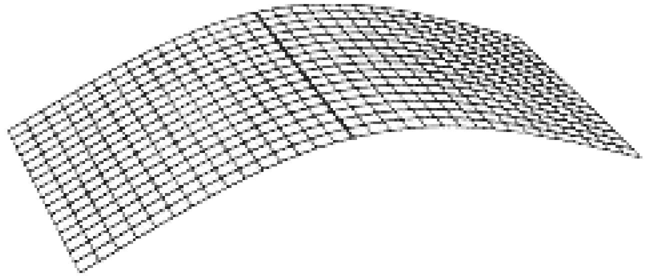


C^0 G^0



C^0 G^1



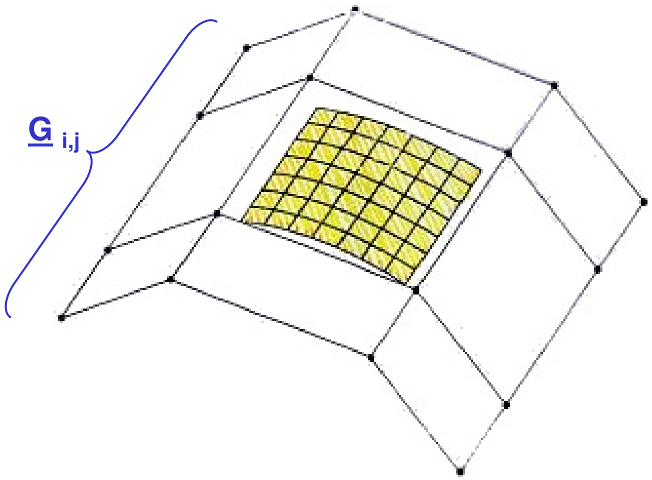


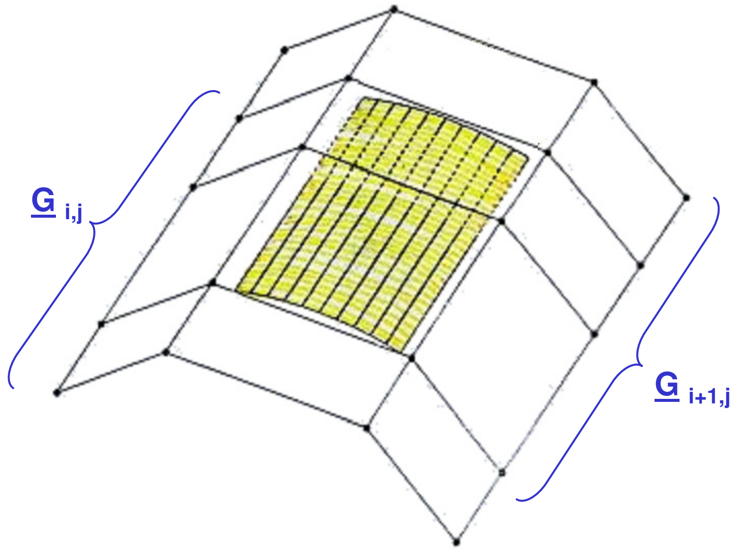
Garante-se C^0G^0 , C^0G^1 ou C^1G^1 por construção geométrica

(via coincidência de pontos, colinearidade e controlo de distâncias)

FORMA de B-SPLINE

$$Q(s,t) = S \cdot M_{Bs} \cdot \underline{G} \cdot M_{Bs}^{T} \cdot T^{T}$$





Garante-se C^2 pela utilização dos pontos de controlo via matriz de geometria $\underline{G}_{i,j}$

à semelhança das curvas B-spline com o vector de geometria G_i

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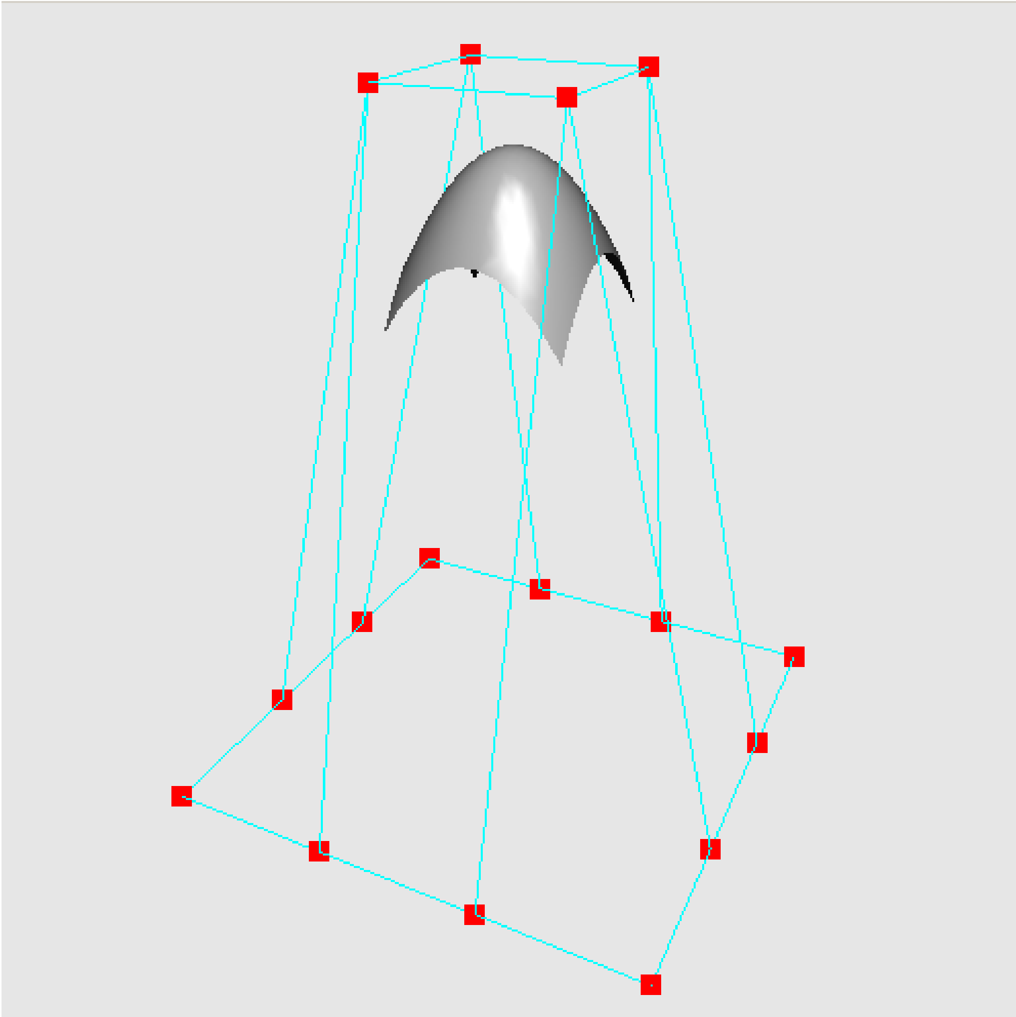
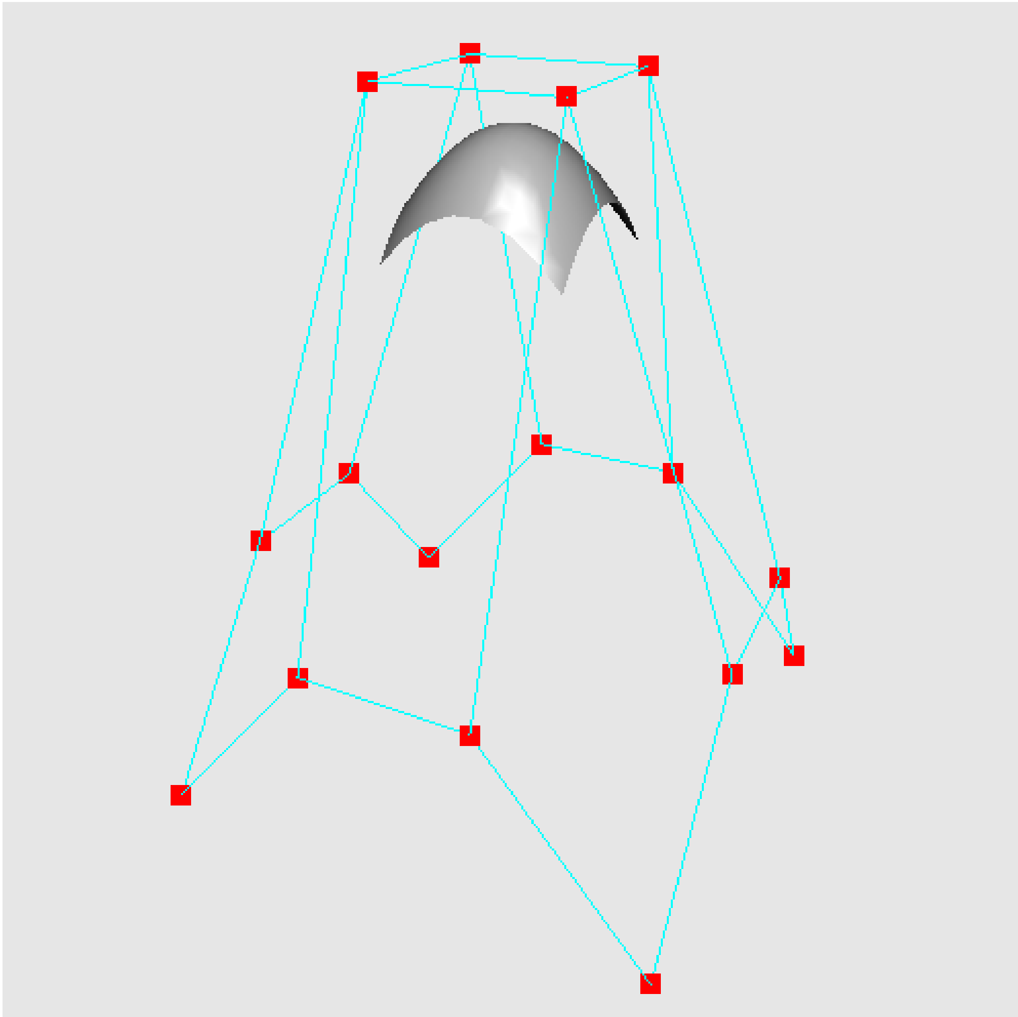
EXEMPLOS



B-spline



B-spline

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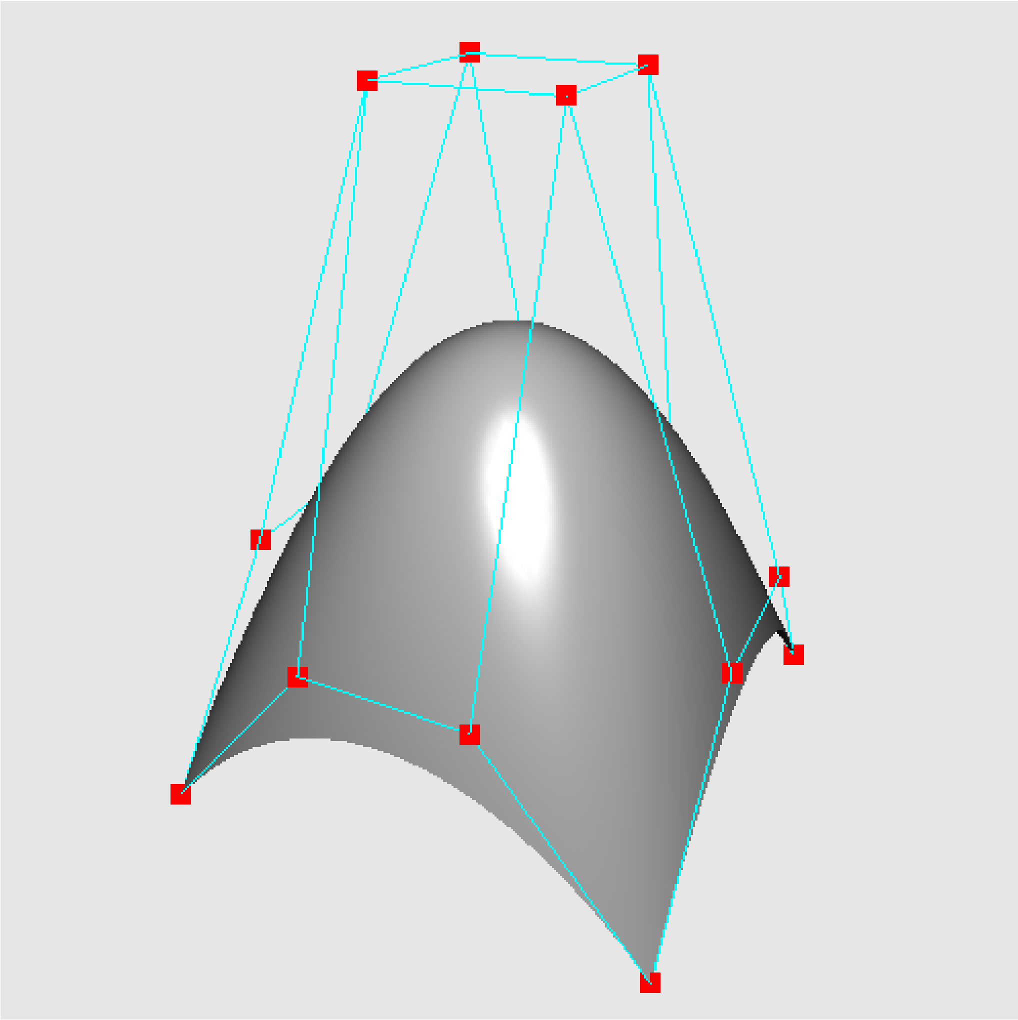

Bézier

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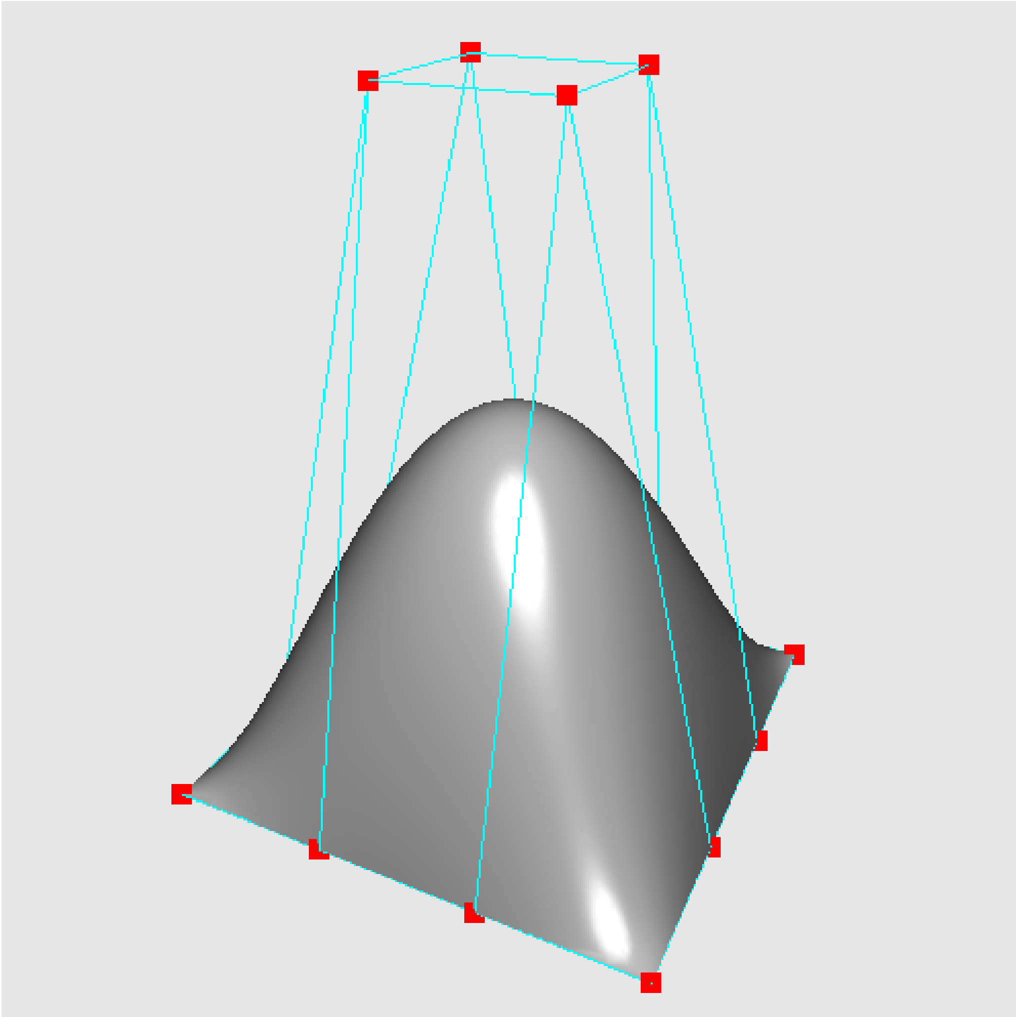

Bézier

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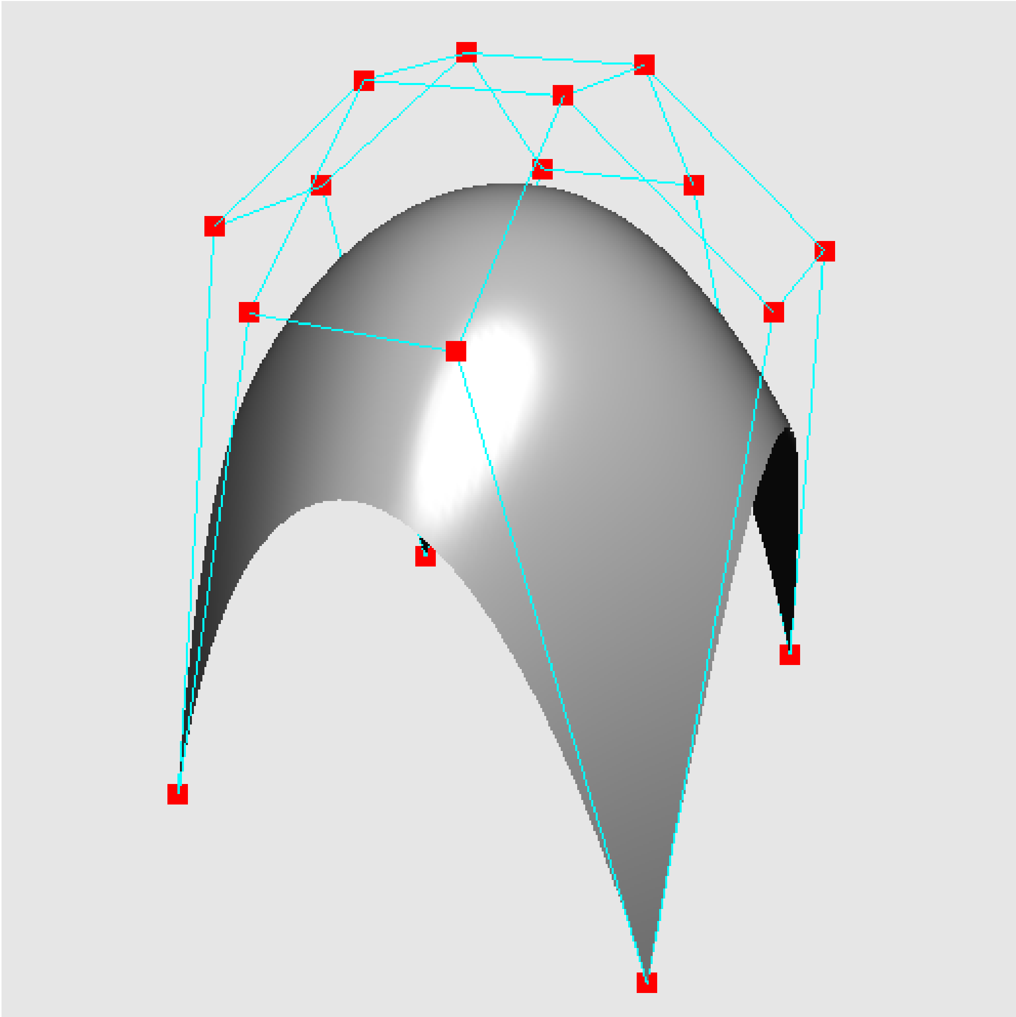

Bézier

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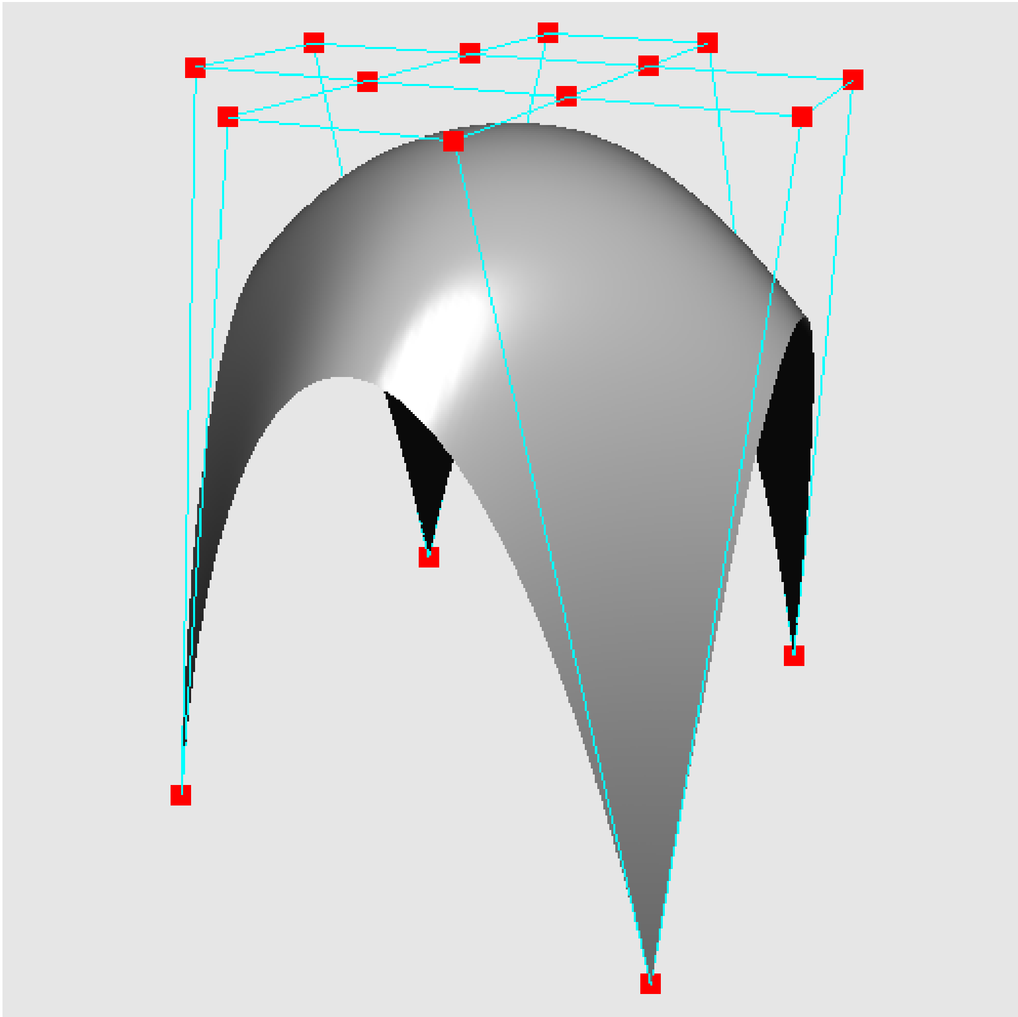


O vetor normal à superfície em cada ponto é imprescindível para a obtenção do sombreamento.

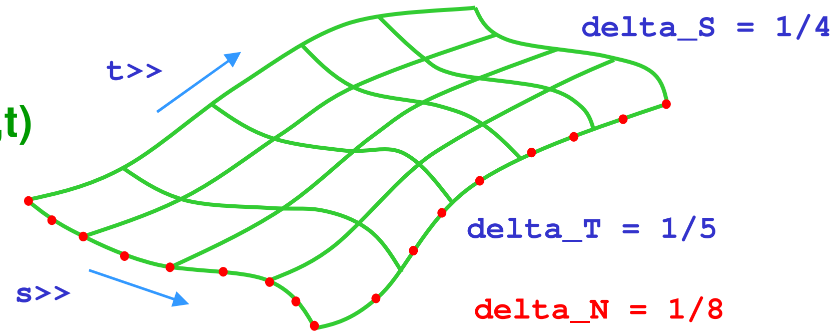
Bézier

AVALIAÇÃO DE UMA SUPERFÍCIE BICÚBICA

Recorde-se que

$$
\begin{aligned}
Q(s,t) = S \,.\, A \,.\, T^T = \; & a_{11} * s^3 * t^3 + a_{12} * s^3 * t^2 + a_{13} * s^3 * t + a_{14} * s^3 + \\
& a_{21} * s^2 * t^3 + a_{22} * s^2 * t^2 + a_{23} * s^2 * t + a_{24} * s^2 + \\
& a_{31} * s * t^3 + a_{32} * s * t^2 + a_{33} * s * t + a_{34} * s + \\
& a_{41} * t^3 + a_{42} * t^2 + a_{43} * t + a_{44}
\end{aligned}
$$

retalho (*patch*) **Q(s,t)**



Algoritmo básico de visualização por curvas:

```
for (patch=first to last)
    for (s=0 to 1 step=delta_S)
        for (t=delta_N to 1 step=delta_N)
            Line_3D( Q(s,t-delta_N), Q(s,t) )
    for (t=0 to 1 step=delta_T)
        for (s=delta_N to 1 step=delta_N)
            Line_3D( Q(s-delta_N,t), Q(s,t) )
```

delta_N ≤ delta_T

delta_N ≤ delta_S

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2D OpenGL Evaluators
applied to bicubic patches

Examples in C

Example:

```
{ a1,a2,a3, b1,b2,b3, c1,c2,c3, d1,d2,d3,
  e1,e2,e3, f1,f2,f3, g1,g2,g3, h1,h2,h3,
  i1,i2,i3, j1,j2,j3, k1,k2,k3, l1,l2,l3,
  m1,m2,m3, n1,n2,n3, o1,o2,o3, p1,p2,p3  }
```

Basic procedure:

1) **Define the evaluator**

```
glMap2d(GL_MAP2_VERTEX_3, 0.0, 1.0,  3, 4,
                          0.0, 1.0, 12, 4, control_points);
```

s stride (3), s order (4), t stride (12), t order (4)

2) **Enable it with**

```
glEnable(GL_MAP2_VERTEX_3);
```

3a) **Invoke it by calling**

```
glEvalCoord2d(s, t);
```

To be used instead of `glvertex3d(` *Q(s,t)* `)`

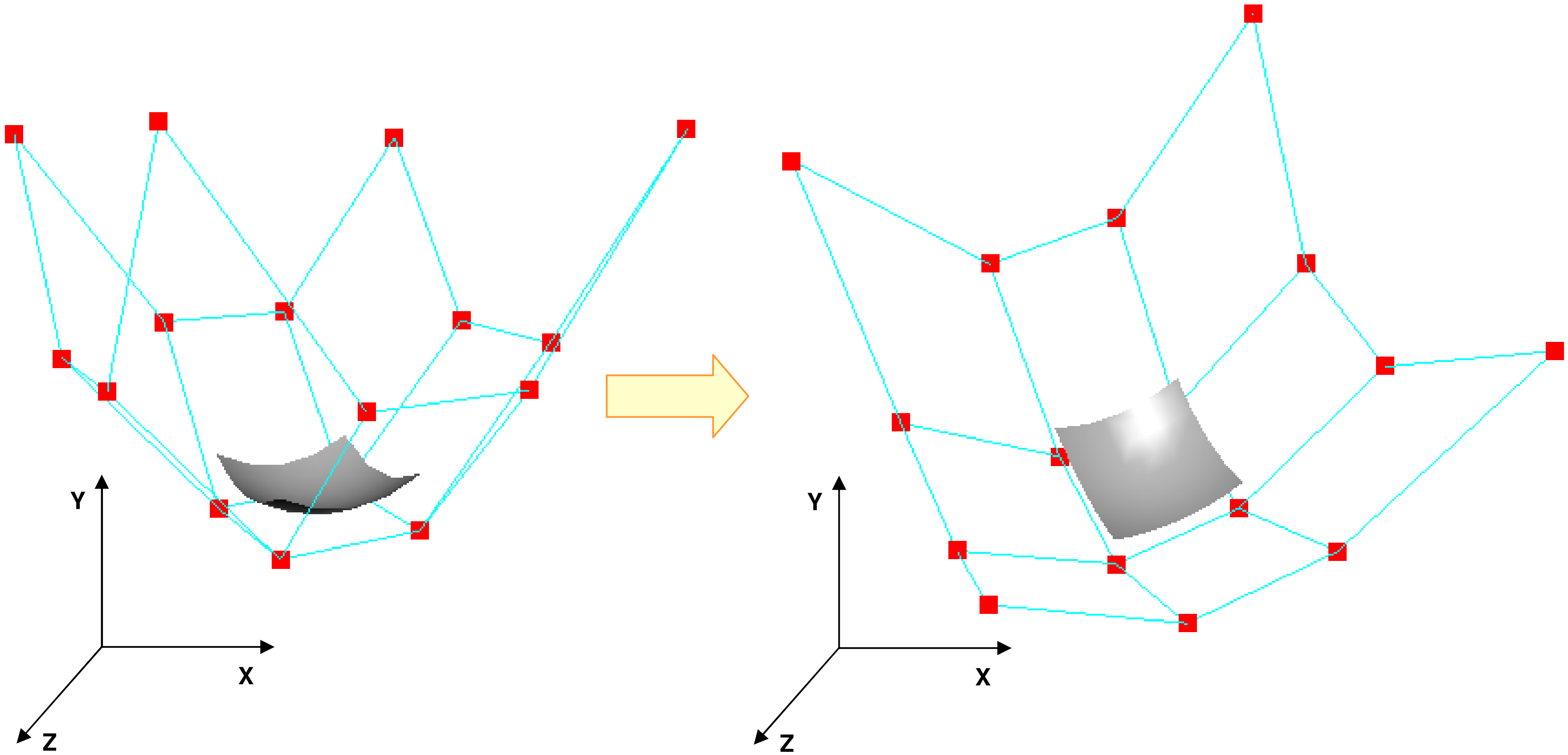
3b) **or**

```
glMapGrid2(1/delta_S, 0.0, 1.0, 1/delta_T, 0.0 , 1.0);
glEvalMesh2(mode, 0, 1/delta_S, 0, 1/delta_T);
```

mode:
`GL_LINE`
(a mesh in wireframe) or
`GL_FILL`
(a filled quad-mesh)

TRANSFORMAÇÕES GEOMÉTRICAS DE CURVAS E SUPERFÍCIES

S, R ou T podem aplicar-se aos coeficientes dos vetores de geometria ou das matrizes de geometria.



(As rotações apenas foram aplicadas aos 16 pontos de controlo e a superfície foi novamente gerada)

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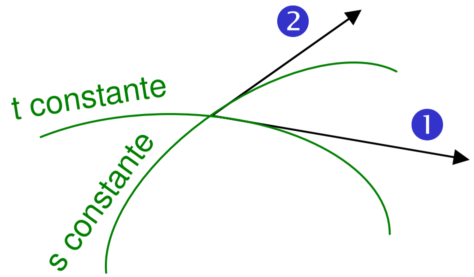
PLANO TANGENTE A UMA SUPERFÍCIE BICÚBICA

❶ **Vetor tangente segundo s :**

$$\frac{\partial}{\partial s} Q(s,t) = \frac{\partial}{\partial s} (S . M . \underline{G} . M^T . T^T)$$

$$= [\ 3s^2 \ \ 2s \ \ 1 \ \ 0\ ] . M . \underline{G} . M^T . T^T$$



❷ **Vetor tangente segundo t :**

$$\frac{\partial}{\partial t} Q(s,t) = \frac{\partial}{\partial t} (S . M . \underline{G} . M^T . T^T)$$

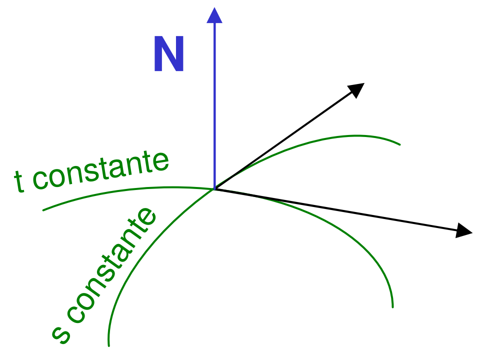
$$= S . M . \underline{G} . M^T . \begin{bmatrix} 3t^2 \\ 2t \\ 1 \\ 0 \end{bmatrix}$$

VETOR NORMAL A UMA SUPERFÍCIE BICÚBICA

Vetor normal :

$$\frac{\partial}{\partial s} Q(s,t) \ \mathrm{X} \ \frac{\partial}{\partial t} Q(s,t) = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ x_s & y_s & z_s \\ x_t & y_t & z_t \end{vmatrix}$$

produto externo



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Polinómios de avaliação dispendiosa

(biquintos = 2 variáveis + grau 5)