Transformações Geométricas 3D

Transformações Geométricas em 3D

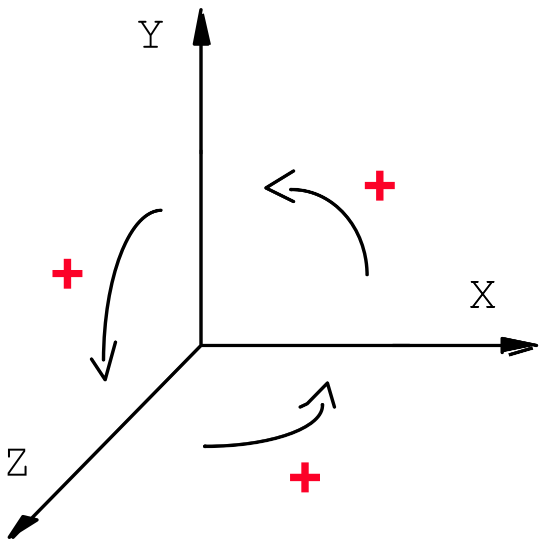
T (x,y,z)

S (Sx,Sy,Sz)

$R_Z(\alpha)$

$R_X(\alpha)$

$R_Y(\alpha)$



Transformações inversas:

T (x,y,z) ⟶ T (-x,-y,-z)

S (Sx,Sy,Sz) ⟶ S (1/Sx,1/Sy,1/Sz)

R (α) ⟶ R (- α)

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Transformações Geométricas em 3D

$$T(T_x, T_y, T_z) = \begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$S(S_x, S_y, S_z) = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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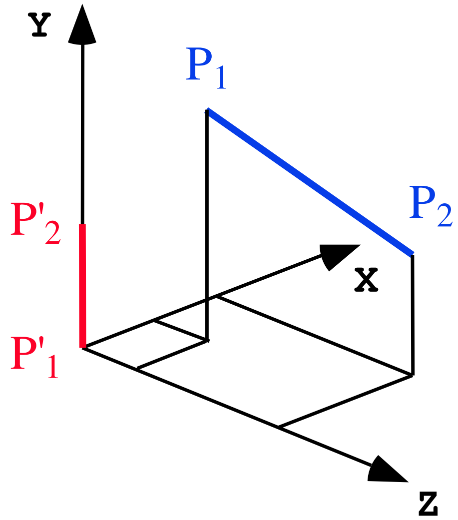
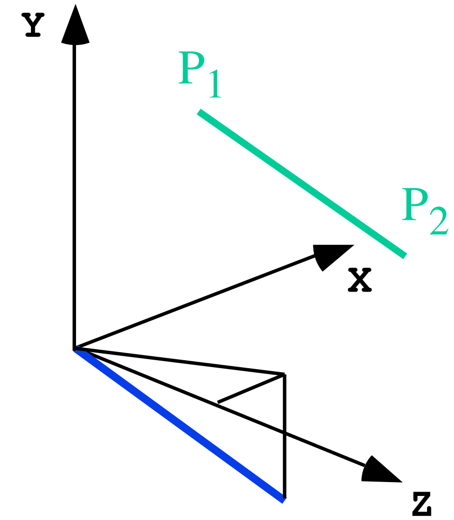
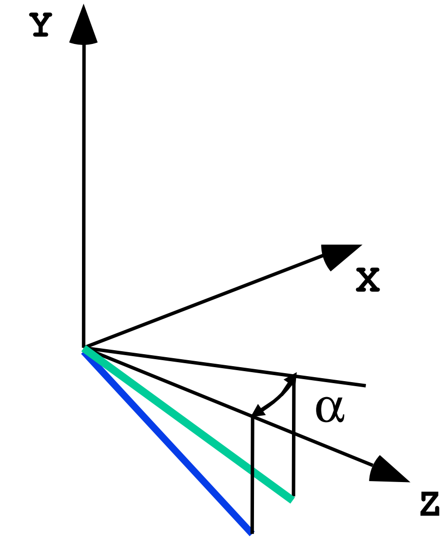
EXEMPLO de APLICAÇÃO 3D, por composição de transformações elementares: Transformar P_1P_2 em $P'_1P'_2$

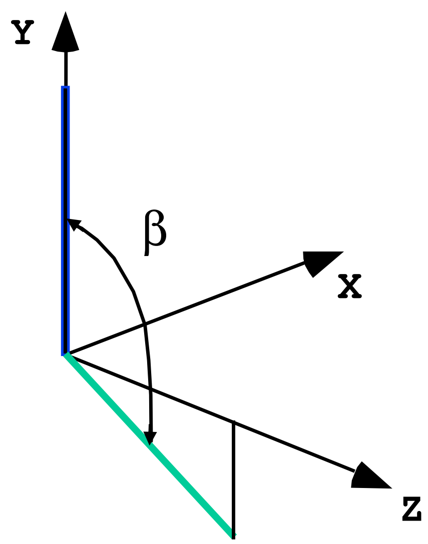




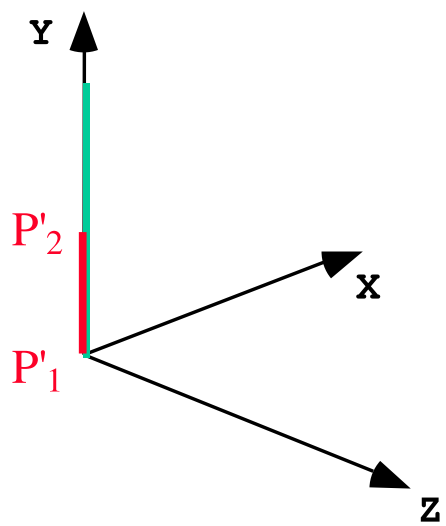
$T(T_X, T_Y, T_Z)$



$R_Y(-\alpha) . T(T_X, T_Y, T_Z)$



$R_x(-\beta) . R_Y(-\alpha) . T(T_X, T_Y, T_Z)$



$S(1, S_Y, 1) . R_x(-\beta) . R_Y(-\alpha) . T(T_X, T_Y, T_Z)$

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3D

Quando é que se pode garantir a comutatividade?

$R_i(\propto).R_i(\beta)$

$S(K1,K2,K3).S(K4,K5,K6)$

$T(D1,D2,D3).T(D4,D5,D6)$

$S(K,K,K3).R_z(\propto)$

$S(K1,K,K).R_x(\propto)$

$S(K,K2,K).R_y(\propto)$

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