Introduction to Computer Graphics with WebGL

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Representation

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Objectives

- Introduce concepts such as dimension and basis
- Introduce coordinate systems for representing vectors spaces and frames for representing affine spaces
- Discuss change of frames and bases

Linear Independence

- A set of vectors $v_1, v_2, \ldots, v_n$ is *linearly independent* if

$$\alpha_1 v_1 + \alpha_2 v_2 + \ldots \alpha_n v_n = 0 \text{ iff } \alpha_1 = \alpha_2 = \ldots = 0$$

- If a set of vectors is linearly independent, we cannot represent one in terms of the others
- If a set of vectors is linearly dependent, at least one can be written in terms of the others

Dimension

- In a vector space, the maximum number of linearly independent vectors is fixed and is called the *dimension* of the space
- In an *n*-dimensional space, any set of n linearly independent vectors form a *basis* for the space
- Given a basis $v_1, v_2, \ldots, v_n$, any vector v can be written as

$$v = \alpha_1 v_1 + \alpha_2 v_2 + \ldots + \alpha_n v_n$$

where the $\{\alpha_i\}$ are unique

Representation

- Until now we have been able to work with geometric entities without using any frame of reference, such as a coordinate system
- Need a frame of reference to relate points and objects to our physical world.
    - For example, where is a point? Can't answer without a reference system
    - World coordinates
    - Camera coordinates

Coordinate Systems

- Consider a basis $v_1, v_2, \ldots, v_n$
- A vector is written $v=\alpha_1 v_1 + \alpha_2 v_2 + \ldots + \alpha_n v_n$
- The list of scalars $\{\alpha_1, \alpha_2, \ldots \alpha_n\}$ is the *representation* of v with respect to the given basis
- We can write the representation as a row or column array of scalars

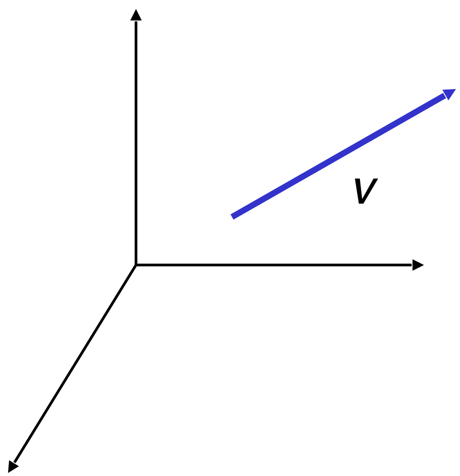
$$\mathbf{a}=[\alpha_1 \;\; \alpha_2 \;\; \ldots \;\; \alpha_n]^T = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \cdot \\ \alpha_n \end{bmatrix}$$

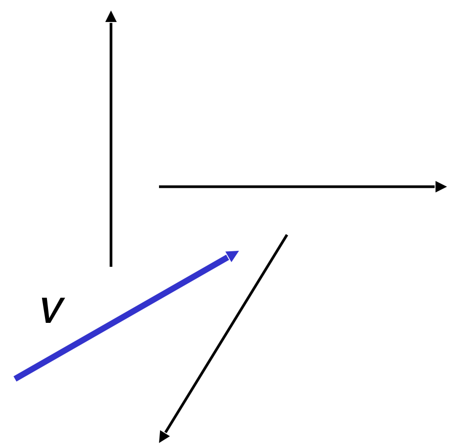
Example

- $v=2v_1+3v_2-4v_3$
- $\mathbf{a}=[2\ 3\ -4]^T$
- Note that this representation is with respect to a particular basis
- For example, in WebGL we will start by representing vectors using the object basis but later the system needs a representation in terms of the camera or eye basis

Coordinate Systems

- Which is correct?

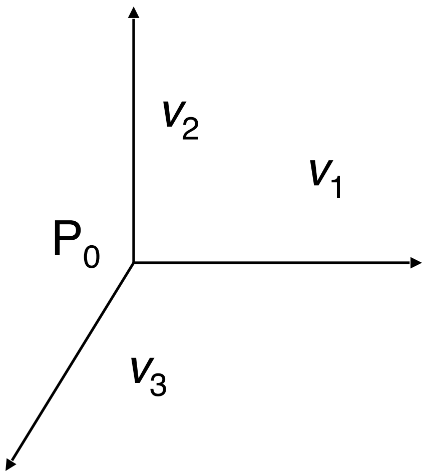




- Both are because vectors have no fixed location

Frames

- A coordinate system is insufficient to represent points
- If we work in an affine space we can add a single point, the *origin*, to the basis vectors to form a *frame*



Representation in a Frame

- Frame determined by (P_0, v_1, v_2, v_3)
- Within this frame, every vector can be written as

$$v = \alpha_1 v_1 + \alpha_2 v_2 + \ldots + \alpha_n v_n$$

- Every point can be written as

$$P = P_0 + \beta_1 v_1 + \beta_2 v_2 + \ldots + \beta_n v_n$$

Confusing Points and Vectors

Consider the point and the vector

$$P = P_0 + \beta_1 v_1 + \beta_2 v_2 + \ldots + \beta_n v_n$$

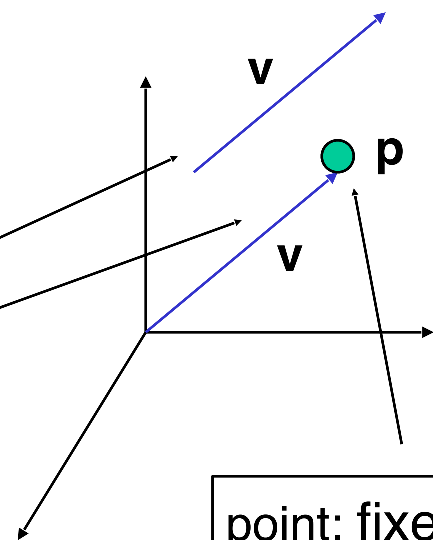
$$v = \alpha_1 v_1 + \alpha_2 v_2 + \ldots + \alpha_n v_n$$

They appear to have the similar representations

p$=[\beta_1\ \beta_2\ \beta_3]$ $\qquad$ **v**$=[\alpha_1\ \alpha_2\ \alpha_3]$

which confuses the point with the vector

A vector has no position



Vector can be placed anywhere

point: fixed