Lógica Computacional

LEI, 2010/2011
FCT UNL

Aulas Práticas 20 e 21

Dedução Natural em Lógica de Primeira Ordem

Prove as seguintes afirmações.

1. $\{\forall x\, P(x) \vee \forall x\, Q(x)\} \vdash \forall x\, (P(x) \vee Q(x))$
2. $\vdash (\forall x\, P(x) \wedge \forall x\, Q(x)) \leftrightarrow \forall x\, (P(x) \wedge Q(x))$
3. $\{\exists x\, (P(x) \wedge Q(x))\} \vdash \exists x\, P(x) \wedge \exists x\, Q(x)$
4. $\vdash (\exists x\, P(x) \vee \exists x\, Q(x)) \leftrightarrow \exists x\, (P(x) \vee Q(x))$
5. $\{\forall x\, (P(x) \rightarrow Q(x))\} \vdash \forall x\, P(x) \rightarrow \forall x\, Q(x)$
6. $\{\exists y\, \forall x\, \varphi\} \vdash \forall x\, \exists y\, \varphi$
7. $\vdash \exists x\, \neg P(x) \leftrightarrow \neg \forall x\, P(x)$
8. $\vdash \forall x\, \neg P(x) \leftrightarrow \neg \exists x\, P(x)$
9. $\vdash \exists x\, \varphi \leftrightarrow \neg \forall x\, \neg \varphi$
10. $\vdash \forall x\, \varphi \leftrightarrow \neg \exists x\, \neg \varphi$
11. $\vdash \forall x\, \varphi \wedge \psi \leftrightarrow \forall x\, (\varphi \wedge \psi)$, se $x \notin \mathsf{VL}(\psi)$
12. $\vdash \forall x\, \varphi \vee \psi \leftrightarrow \forall x\, (\varphi \vee \psi)$, se $x \notin \mathsf{VL}(\psi)$
13. $\vdash \exists x\, \varphi \wedge \psi \leftrightarrow \exists x\, (\varphi \wedge \psi)$, se $x \notin \mathsf{VL}(\psi)$
14. $\vdash \exists x\, \varphi \vee \psi \leftrightarrow \exists x\, (\varphi \vee \psi)$, se $x \notin \mathsf{VL}(\psi)$
15. $\vdash \forall x\, (\psi \rightarrow \varphi) \leftrightarrow \psi \rightarrow \forall x\, \varphi$, se $x \notin \mathsf{VL}(\psi)$
16. $\vdash \exists x\, (\psi \rightarrow \varphi) \leftrightarrow \psi \rightarrow \exists x\, \varphi$, se $x \notin \mathsf{VL}(\psi)$
17. $\vdash \forall x\, (\varphi \rightarrow \psi) \leftrightarrow \exists x\, \varphi \rightarrow \psi$, se $x \notin \mathsf{VL}(\psi)$
18. $\vdash \exists x\, (\varphi \rightarrow \psi) \leftrightarrow \forall x\, \varphi \rightarrow \psi$, se $x \notin \mathsf{VL}(\psi)$