A 1 a) $k = ?$

$$\int_{-\infty}^{+\infty} f_x(x)\,dx = 1 \iff \int_{-1}^{0} k + x\,dx + \int_{0}^{1} k - x\,dx = 1$$

$$\iff \left[kx + \frac{x^2}{2}\right]_{-1}^{0} + \left[kx - \frac{x^2}{2}\right]_{0}^{1} = 1 \iff 0 - \left(-k + \frac{1}{2}\right) + \left(k - \frac{1}{2}\right) = 1$$

$$\iff k - \frac{1}{2} + k - \frac{1}{2} = 1 \iff 2k - 1 = 1 \iff k = 1$$

b) $F_x(x) = ?$

$\boxed{x < -1}$ $F_x(x) = 0$

$\boxed{0 \leq x < 1}$ $F_x(x) = \int_{-\infty}^{x} f_x(t)\,dt = \int_{-\infty}^{-1} 0\,dt + \int_{-1}^{0} (1+t)\,dt + \int_{0}^{x} (1-t)\,dt$

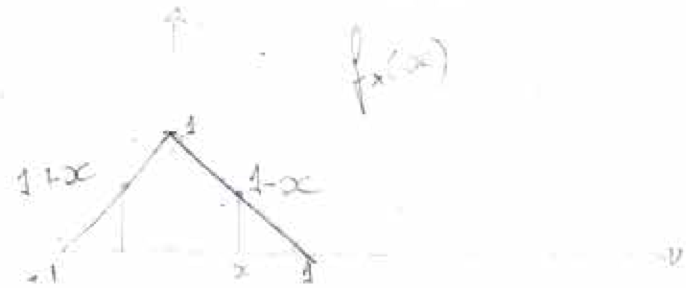
$$= \left[t + \frac{t^2}{2}\right]_{-1}^{0} + \left[t - \frac{t^2}{2}\right]_{0}^{x} = +1 - \frac{1}{2} + x - \frac{x^2}{2} = \frac{2x - x^2}{2} + \frac{1}{2}$$

$$= \frac{1 + 2x - x^2}{2}$$

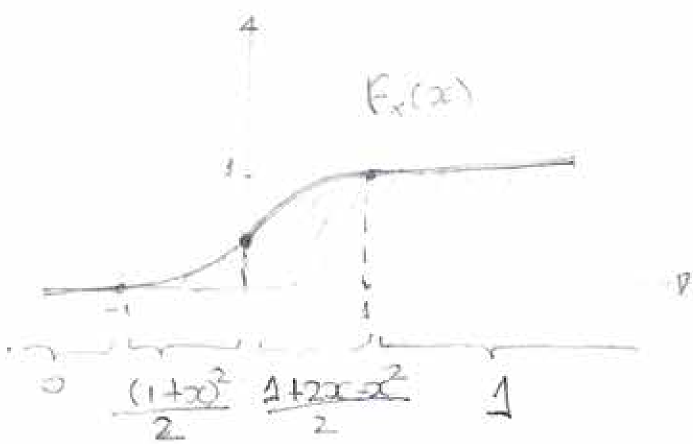
$\boxed{-1 \leq x < 0}$ $F_x(x) = \int_{-\infty}^{x} f_x(t)\,dt = \cancel{\int_{-\infty}^{-1} 0\,dt} + \int_{-1}^{x} f_x(t)\,dt = \int_{-1}^{x} 1 + t\,dt$

$$= \left[t + \frac{t^2}{2}\right]_{-1}^{x} = x + \frac{x^2}{2} + 1 - \frac{1}{2} = x + \frac{x^2}{2} + \frac{1}{2} = \frac{x^2 + 2x + 1}{2} = \frac{(1+x)^2}{2}$$

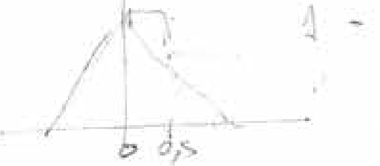
$\boxed{x > 1}$ $F_x(x) = \int_{-\infty}^{x} f_x(t)\,dt = 1$



$$F_x(x) = \begin{cases} 0 & \text{se } x < -1 \\ \frac{(1+x)^2}{2} & \text{se } -1 \leq x < 0 \\ \frac{1+2x-x^2}{2} & \text{se } 0 \leq x < 1 \\ 1 & \text{se } x > 1 \end{cases}$$



c) $P(X>0) = ?$



$= 1 - P(X \leq 0)$

$= 1 - \int_{-\infty}^{0} f_X(x)\,dx$ ou $= 1 - F(0) = 1 - \frac{1}{2} = \frac{1}{2}$ //

d) $P(X>0{,}5 \mid X>0) = \dfrac{P[(X>0{,}5) \cap (X>0)]}{P(X>0)}$

$= \dfrac{P(X>0{,}5)}{P(X>0)} \rightarrow \dfrac{1 - P(x<0{,}5)}{1 - F_X(0)} = \dfrac{1 - F_X(0{,}5)}{\frac{1}{2}} =$

$\dfrac{1 - (1 - \frac{1}{8})}{\frac{1}{2}} = \boxed{F_X(\tfrac{1}{2}) = \tfrac{1}{2} - (\tfrac{1}{2}\,\tfrac{1}{4}) + \tfrac{1}{2} = 1 - \tfrac{1}{8}} = \frac{1}{4}$

e) $\mathbb{E}[X] = 0$ pela região simétrica $\boxed{= \int_{-\infty}^{+\infty} x f_X(x)\,dx}$

$= \int_{-1}^{0} x(1+x)\,dx + \int_{0}^{1} x(1-x)\,dx =$

$= \int_{-1}^{0} (x + x^2)\,dx + \int_{0}^{1} (x - x^2)\,dx = \left[\frac{x^2}{2} + \frac{x^3}{3}\right]_{-1}^{0} + \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_{0}^{1} =$

$= -\left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) = 0$ //

$\boxed{\mathbb{V}[X] = \mathbb{E}[X^2] - (\cancel{\mathbb{E}[X]})^2}$, $\mathbb{E}[X] = 0$

$\mathbb{V}[X] = \int_{-1}^{0} x^2(1+x)\,dx + \int_{0}^{1} x^2(1-x)\,dx = \int_{-1}^{0} (x^2 + x^3)\,dx +$

$+ \int_{0}^{1} (x^2 - x^3)\,dx = \left[\frac{x^3}{3} + \frac{x^4}{4}\right]_{-1}^{0} + \left[\frac{x^3}{3} - \frac{x^4}{4}\right]_{0}^{1} = -\left(-\frac{1}{3} + \frac{1}{4}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) =$

$= 2\left(\frac{1}{3} - \frac{1}{4}\right) = \frac{1}{6}$ //

2 A a)

$$\int_{-\infty}^{+\infty} f_x(x)\,dx = 1 \Rightarrow \int_0^k 4x\,dx = 1 \Leftrightarrow \left[2x^2\right]_0^k = 1$$

$$\Leftrightarrow 2k^2 = 1 \Rightarrow k^2 = \frac{1}{2} \Rightarrow k = \sqrt{1/2} = \frac{1}{\sqrt{2}}$$

b) $F_x(x) = ?$ $\quad 0 < x < \frac{1}{\sqrt{2}}$

$$\int_{-\infty}^{x} f_x(t)\,dt = \int_{-\infty}^{0} 0\,dt + \int_0^x 4t\,dt = \left[2t^2\right]_0^x = 2x^2$$

$$F_x(x) = \begin{cases} 0 & , x < 0 \\ 2x^2 & 0 \leq x < \frac{1}{\sqrt{2}} \\ 1 & x \geq \frac{1}{\sqrt{2}} \end{cases}$$

c) $P(\frac{1}{4} \leq X \leq \frac{1}{3})$

$$= \int_{\frac{1}{3}}^{\frac{1}{4}} 4x\,dx = \left[2x^2\right]_{\frac{1}{4}}^{\frac{1}{3}} = \frac{2}{9_{(8)}} - \frac{1}{8_{(9)}} = \frac{16-9}{72} = \frac{7}{72}$$

d) $$E[X] = \int_{-\infty}^{+\infty} x f_x(x)\,dx \quad \int_0^{\frac{1}{\sqrt{2}}} 4x^2\,dx = \left[\frac{4x^3}{3}\right]_0^{\frac{1}{\sqrt{2}}} = \frac{4 \times \frac{1}{2\sqrt{2}}}{3} = 0{,}4714$$

$$V[X] = E[X^2] - (E[X])^2$$

$$\hookrightarrow \int_{-\infty}^{+\infty} x^2 f_x(x)\,dx = \int_0^{\frac{1}{\sqrt{2}}} x^2 \cdot 4x\,dx = \left[x^4\right]_0^{\frac{1}{\sqrt{2}}}$$

$$= \frac{1}{\sqrt{2}}^4 - 0 = \frac{1}{4}$$

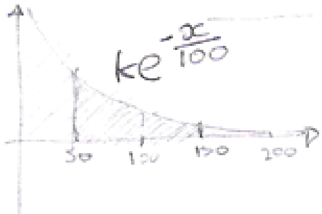
$$V[X] = \frac{1}{4} - (0{,}4714)^2 = 0{,}02778$$

A3) $\int_{-\infty}^{+\infty} f_E(t)\,dt = 1 \Leftrightarrow \int_{0}^{100} k e^{-\frac{t}{100}}\,dt = 1 \Leftrightarrow k\int_{0}^{+\infty} e^{-\frac{t}{100}} = 1 \Leftrightarrow k\left[-100 e^{-\frac{t}{100}}\right]_{0}^{+\infty} = 1$

$\Leftrightarrow k \times -100 \times e^{0} - 100k = 1 \Leftrightarrow -100k \times (0-1) = 1 \Rightarrow 100k = 1 \Leftrightarrow k = \frac{1}{100}$ //

$F_E(x) = \int_{-\infty}^{x} f_E(t)\,dt = \int_{0}^{x} \frac{1}{100} e^{-\frac{t}{100}}\,dt = \left[e^{-\frac{t}{100}}\right]_{x}^{0} = 1 - e^{-\frac{x}{100}}$ → Demonstração Apontamentos das aulas.

$F_E(x) = \begin{cases} 0, & x < 0 \\ 1 - e^{-\frac{x}{100}}, & x \geq 0 \end{cases}$



a) $P(50 \leq X \leq 150) = F(150) - F(50) = 0{,}77687 - 0{,}39347$

$= 0{,}3834$ //

b) $P(X < 100) = F(100) = 1 - e^{-\frac{100}{100}} = 0{,}63212$

$P(X = 100) = \emptyset$?

c) $P(X > 200 \mid X > 100) = \frac{P((X>200) \wedge (X>100))}{P(X>100)} = \frac{P(X>200)}{P(X>100)}$

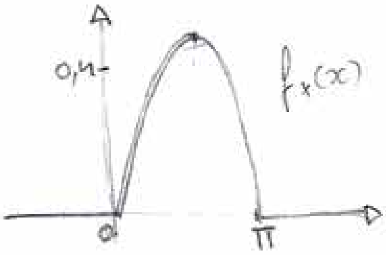
$= \frac{1 - P(X \leq 200)}{1 - P(X \leq 100)} = \frac{1 - (1 - e^{-2})}{1 - 0{,}63212} = \frac{0{,}13533}{0{,}36788} = 0{,}3679$ //

d) $E[X] = ?$ $\lambda = 0$ $\delta = 100$

$E[X] = \lambda + \delta = 100$

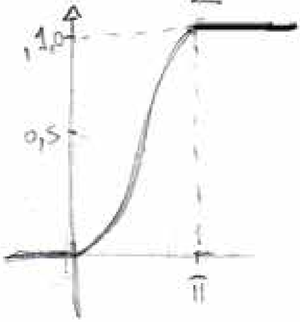
$V[X] = \delta^{2} = 10.000$

A4



a) $F_X(x) = ?$ $\displaystyle\int_0^x f_X(t)\,dt = \int_0^x \frac{\operatorname{sen}(t)}{2}\,dt = \left[-\frac{\cos(t)}{2}\right]_0^x$

$\displaystyle = -\frac{\cos(x)}{2} + \frac{1}{2} = \frac{1-\cos(x)}{2}$ $\quad F_X(x) = \begin{cases} 0 & \text{se } X<0 \\ \dfrac{1-\cos(x)}{2} & \text{se } 0 \leq X \leq \pi \\ 1 & \text{se } X > \pi \end{cases}$

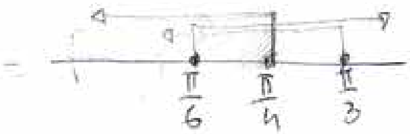


b) $P(X \leq a) = P(X \geq a)$

$\displaystyle \frac{1-\cos(a)}{2} = 0{,}5 \Rightarrow 1-\cos a = 1 \Rightarrow -\cos(a) = 0$

$\Rightarrow a = \frac{\pi}{2}$ //

c) $\displaystyle P\left(X \leq \frac{\pi}{4} \,\middle|\, \frac{\pi}{6} < X < \frac{\pi}{3}\right) = \frac{P\left[\left(X \leq \frac{\pi}{4}\right) \cap \left(\frac{\pi}{6} < X < \frac{\pi}{3}\right)\right]}{P\left(\frac{\pi}{6} < X < \frac{\pi}{3}\right)}$



$\displaystyle = \frac{P\left(\frac{\pi}{6} < X < \frac{\pi}{4}\right)}{P\left(\frac{\pi}{6} < X < \frac{\pi}{3}\right)} = \frac{F\left(\frac{\pi}{4}\right) - F\left(\frac{\pi}{6}\right)}{F\left(\frac{\pi}{3}\right) - F\left(\frac{\pi}{6}\right)} = \frac{\left(1-\frac{1}{\sqrt{2}}\right) - \left(1-\frac{\sqrt{3}}{2}\right)}{\left(1-\frac{1}{2}\right) - \left(1-\frac{\sqrt{3}}{2}\right)} = \frac{0.1589}{0.3660}$

$= 0{,}4341$ //

d) $\displaystyle \int_{-\infty}^{+\infty} x f_X(x)\,dx = \int_0^{\pi} \frac{x \cdot \operatorname{sen}(x)}{2}\,dx = \left[\frac{\operatorname{sen} x - x \cdot \cos x}{2}\right]_0^{\pi}$

$\displaystyle = \frac{0 - \pi \times (-1)}{2} - 0 = \frac{\pi}{2}$ // $\quad E[X] = \frac{\pi}{2}$ //

$\displaystyle V[X] = E[X^2] - E[X]^2 = 2{,}9348 - \left(\frac{\pi}{2}\right)^2 = 2{,}9348 - 2{,}4674 = 0{,}46739$ //

$\displaystyle \hookrightarrow \int_{-\infty}^{+\infty} \frac{x^2 \operatorname{sen} x}{2}\,dx = \left[-\frac{1}{2}x^2 \cos x + x \operatorname{sen} x + \cos x\right]_0^{\pi}$

$\displaystyle = \left(-\frac{\pi^2}{2} \times (-1) + 0 + (-1)\right) - (0 + 0 + 1) = \frac{\pi^2}{2} - 2 = \frac{\pi^2 - 4}{2} = 2{,}9348$

A5

a) $k = ?$

$$\int_{-\infty}^{+\infty} f_X(x)\,dx = 1 \iff \int_0^{\pi} k \operatorname{sen} x \, dx = 1 \iff k\left[-\cos x\right]_0^{\pi} = 1$$

$$\iff k \times (-\cos\pi + \cos 0) = 1 \iff k = \frac{1}{1+1} \iff k = \frac{1}{2}$$ //

b) $\mathbb{E}[X] = ?$ $$\int_{-\infty}^{+\infty} x f x \, dx = \int_0^{\pi} \frac{x \operatorname{sen} x}{2} dx = \left[\frac{\operatorname{sen} x - x\cos x}{2}\right]_0^{\pi}$$

$$= \frac{\operatorname{sen}\pi - \pi\times\cos\pi}{2} - \frac{\operatorname{sen} 0 - 0}{2} = \frac{0+\pi}{2} - 0 = \frac{\pi}{2}$$ //

c) $$\mathbb{V}[X] = \mathbb{E}[X^2] - \mathbb{E}[X]^2 \iff \frac{\pi^2}{4} - 2 = \mathbb{E}[X^2] - \left(\frac{\pi}{2}\right)^2$$

$$\Rightarrow \mathbb{E}[X^2] = \frac{\pi^2}{4} - 2 + \frac{\pi^2}{4} = \frac{\pi^2}{2} - 2$$

d) $$\underrightarrow{\mathbb{E}[\cos(X)]} = \cos\left[\mathbb{E}[X]\right] = \cos\frac{\pi}{2} = 0$$

COS FUNÇÃO CONTINUA

OU: $$\int_0^{\pi} \cos x \cdot \frac{1}{2} \sin x \, dx = \frac{1}{2}\left[\frac{(\sin x)^2}{2}\right]_0^{\pi} = \frac{1}{2}(0-0) = 0$$

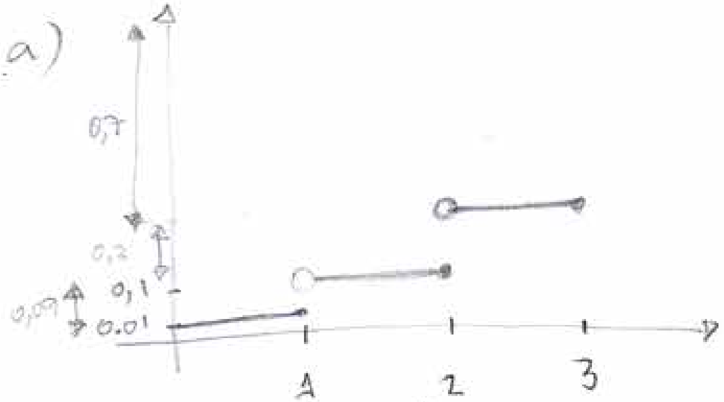
e) $\mathbb{V}(5X-4) \rightarrow$ PROP. VARIANCIA $\mathbb{V}[aX-b] = a^2 \mathbb{V}_X$

$$= 5^2\left(\frac{\pi^2}{2} - 2\right) = \frac{25\pi^2}{2} - 50$$

B6

k	0	1	2	3
$\mathbb{P}[X=k]$	0,01	0,09	0,2	0,7

a)



$$F_X(x) = \begin{cases} 0 & , x \leq 0 \\ 0{,}01 & 0 \leq x < 1 \\ 0{,}1 & 1 \leq x < 2 \\ 0{,}3 & 2 \leq x < 3 \\ 1 & x \geq 3 \end{cases}$$

NB: F_X "salta" nos pontos em que toma valores e a "altura do salto" corresponde à prob. de tomar o valor em que está a saltar.

F_X é contínua à direita

b) Seja u uma observação de $U \sim \mathcal{U}([0,1])$, sabemos que $\mathbb{P}[U \in [a,b]] = b-a$ para qualquer $[a,b] \subseteq [0,1]$

logo a prob. de $u \in [0;0{,}01[$ é 0,01 se for $x=0$ $\mathbb{P}[X=0]=0{,}01$

$u \in [0{,}01;0{,}1[$ é 0,09 " $x=1$ $\mathbb{P}[X=1]=0{,}09$

$u \in [0{,}1;0{,}3[$ é 0,2 ; " $x=2$ $\mathbb{P}[X=2]=0{,}2$

$u \in [0{,}3;1[$ é 0,7 " $x=3$ $\mathbb{P}[X=3]=0{,}7$

função prob X

c) Se $u_1=0{,}2235$ $u_2=0{,}4515$ são NPA $\mathcal{U}[0,1]$

$u_1 \rightsquigarrow x=2$ $u_2 \rightsquigarrow 3$ (de acordo com b)

C1 a) $X \to$ "nº de computadores com software ilegal"

$X \sim H(N=50, M=7, n=6)$

b) $\mathbb{P}(X \geq 1) = 1 - \mathbb{P}(X<1) = 1 - \mathbb{P}(X=0)$

$$\mathbb{P}(X=0)\ f_X(x) = \frac{{}^{7}C_0 \times {}^{50-7}C_{6-0}}{{}^{50}C_6} = 0{,}384$$

logo: $1 - 0{,}384 = 0{,}616$

c) $$\mathbb{P}(X=2) = f_X(2) = \frac{{}^{7}C_2 \times {}^{43}C_4}{{}^{50}C_6} = 0{,}1631$$

d) $\mathbb{P}(X=4) = f_X(4)$

$$= \frac{{}^{7}C_4 \times {}^{43}C_2}{{}^{50}C_6} = 0{,}00191$$

e) $$\mathbb{E}[X] = n \times \frac{M}{N} = 6 \times \frac{7}{50} = 0{,}84$$

f) $$V[X] = n \times \frac{M}{N} \times \frac{N-M}{N} \times \frac{N-M}{N-1} = 0{,}84 \times \frac{43}{50} \times \frac{43}{49} = 58{,}86$$

C2 a) $X \to$ "nº de frangos em bom estado após 10 infecções"

$X \sim H(N=100, M=85, n=10)$

b) $$\mathbb{P}(X=7) = f_X(7) = \frac{{}^{85}C_7 \, {}^{15}C_3}{{}^{100}C_{10}} = 0{,}13$$

c) $\mathbb{P}(X \geq 8) = \mathbb{P}(X=8) \wedge (X=9) \wedge (X=10) = 0{,}29 + 0{,}357 + 0{,}18 = 0{,}8277$

d) $X \sim H(N=100), M=85, n=5)$

$$\mathbb{P}(X=5) = f_X(5) = \frac{{}^{85}C_5 \, {}^{15}C_0}{{}^{100}C_5} = 0{,}4356$$

e) $\mathbb{E}[X] = n \times \frac{M}{N} = 10 \times \frac{85}{100} = 8{,}5$ f) $1 - 8{,}5 = 1{,}5$

(C3) a) $X \to$ "nº de pessoas q fazem refeição na planura em cada 10"

$X \sim Bi(n=10,\ \theta=0{,}2)$

b) $P(X=0) = {}^{n}C_{x} \times \theta^{x} \times (1-\theta)^{n-x} = {}^{10}C_{0} \times 0{,}2^{0} \times 0{,}8^{10} = 0{,}10737$ //

c) $P(X=10) = {}^{10}C_{10} \times 0{,}2^{10} \times 0{,}8^{0} = 0{,}000000102$ //

d) $P(X \geq 1) = 1 - P(X=0) = 0{,}89263$ //

e) $E[X] = n \times \theta = 10 \times 0{,}2 = 2$ //

(C4) a) $X \to$ "nº de clips c/ defeito, em 50 selecionados"

$X \sim Bi(n=50,\ 0{,}05)$

b) $P(X=0) = {}^{50}C_{0} \times 0.05^{0} \times 0.95^{50} = 0.07695$ //

c) $P(X=1) = {}^{50}C_{1} \times 0.05^{1} \times 0.95^{49} = 0.20249$ //

d) $P(X \leq 1) = P(X=0) + P(X=1) = 0.2794$ //

e) $E[X] = n \times \theta = 50 \times 0.05 = 2{,}5$ //

$V[X] = n \times \theta(1-\theta) = 2{,}5 \times (0.95) = 2{,}375$

$\sigma = \sqrt{V[X]} = \sqrt{2{,}375} = 1.5411$ //

(C5) $X \sim (\lambda)$, $\lambda = ?$

$P(X=0) = 0{,}15 \Rightarrow \dfrac{e^{-\lambda} \cdot \lambda^{0}}{0!} = 0{,}15 \Rightarrow e^{-\lambda} = 0{,}15 \xrightarrow{x = \ln 0{,}15} \lambda = 1.89712$

a) $P(X=1) = \dfrac{e^{-1{,}9} \times 1{,}9^{1}}{1!} = 0{,}284$ //

b) $P(X=2) = \dfrac{e^{-1{,}9} \times 1{,}9^{2}}{2!} = 0{,}2699$ //

c) $P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$P(X=3) = \dfrac{e^{-1{,}9} \times 1{,}9^{3}}{3!} = 0{,}1709$, logo $P(X \leq 3) = 0{,}15 + 0{,}284 + 0{,}2699 + 0{,}17 = 0{,}875$ //

d) $P(X \geq 4) = 1 - P(X \leq 3) = 1 - 0{,}875 = 0{,}125$ //

e) $E[X] = \lambda = 1{,}9$, $V[X] = \lambda = 1{,}9$ //

C6) $X \to$ "nº pessoas que compram no site em cada dia" $X \sim P(10)$

$Y \to$ "nº pessoas que compram em uma semana"

a) $Y \sim P(\lambda = 7\times 10) = P(\lambda = 70)$

* a média da distr. Poisson é directamente proporcional à amplitude do intervalo.

$P(Y=0) = \frac{e^{-70} \times 70^0}{0!} = 3,975\times 10^{-31}$ //

b) $P(X=50) = \frac{e^{-10} \times 10^{50}}{50!} = 1,492\times 10^{-19}$

c) $E[Y] = \lambda = 70$, $V[Y] = \lambda = 70$ //

D1) $X \to$ "tempo em horas q um computador está ligado"

Com distrib. uniforme no intervalo $[9,12]$

a) $X \sim U(9,12)$

$$f_X(x) = \begin{cases} \frac{1}{12-9} = \frac{1}{3} & , x \in [9,12] \\ 0 & , x \notin [9,12] \end{cases}$$

b) $E[X] = \frac{a+b}{2} = \frac{21}{2} = 10,5$ $V[X] = \frac{(b-a)^2}{12} = \frac{9}{12} = \frac{1}{3}$ //

c) $P(X \geq a) = 0,90 \Leftrightarrow 1 - P(X < a) = 0,90 \Leftrightarrow$

$1 - \int_9^a \frac{1}{3} dx = 0,90 \Leftrightarrow 1 - \left[\frac{1}{3}x\right]_9^a = 0,90 \Leftrightarrow$

... $a = 1,0333$

d) $X \to$ "PROB > 11h NUM DIA"

$Y \to$ "Nº DE DIAS Q $X > 11$, NUMA SEMANA" $\sim Bi(7, P(X > 11))$

TEMOS Q RESPONDER A PERGUNTA

$P(Y = 5)$

D2 X — o tempo q uma aplicação leva a processar cada ficheiro

$\mathbb{E}[X] = \delta = 3$ m $X \sim E(\lambda, \delta)$

$\hookrightarrow \sqrt{\delta^2} = 3 \Rightarrow \delta = 3$

$\hookrightarrow \lambda + \delta = 3 \Rightarrow \lambda = 0$, logo $X \sim E(0,3)$

a) $F_X(x) = \begin{cases} 0 & , x < 0 \\ 1 - e^{-\frac{x}{3}} & , x \geq 0 \end{cases}$ $f_X(x) = \begin{cases} 0 & , x < 0 \\ \frac{1}{3} e^{-\frac{x}{3}} & , x \geq 0 \end{cases}$

b) $P(x > 5) = 1 - P(x \leq 5) = 1 - F(5) = 1 - 0{,}81112 = 0{,}18887$ //

c) $P(x > 3) = 1 - P(x \leq 3) = 1 - F(3) = 1 - 0{,}6321 = 0{,}3678$ //

d) $P(x > 5 | x > 2) = \frac{P(x>5)}{P(x>2)} = \frac{0{,}18887}{0{,}5134} = 0{,}3678$ //

$P(x>2) = 1 - F(2)$

$= 1 - 0{,}4865 = 0{,}5134$

e) $V = \delta^2 = 9$ $\sigma = \sqrt{V[X]} = \sqrt{9} = 3$

D3 $\lambda = 0$ $\delta = 1000$

$F_X(x) = \begin{cases} 0 & \text{se } x < 0 \\ 1 - e^{-\frac{x}{1000}} & \text{se } x \geq 0 \end{cases}$

a) $P(x < 15000) = F(15000) = 1 - e^{-1{,}5} = 0{,}7768$

b) $\mathbb{E}[X] = \lambda + \delta = 1000$ $\sigma = \sqrt{V[X]} \Leftrightarrow \sigma = \sqrt{1000^2} \Rightarrow \sigma = 1000$

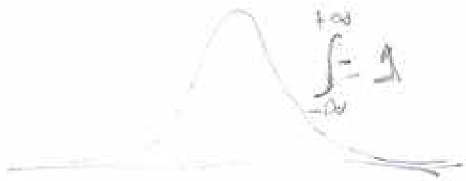
c) $P(x < 15000)$ = avaria dentro da garantia $= 0{,}7768$

logo: $X \sim Bi(n = 20, p = 0{,}7768)$

$P(x=5) = {}^{20}C_5 \times 0{,}7768^5 \times 0{,}2232^{15} = 0{,}000000745$

P4) $X \sim N(90, 12^2)$ $\quad \sigma = 12 \Rightarrow V[X] = 144$

$f_X(x) = \frac{1}{\sqrt{2\pi \times 144}} e^{\left(-\frac{1}{2}\times\left(\frac{x-90}{144}\right)^2\right)}$



$\int_{-\infty}^{+\infty} = 1$

$F_X(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi \times 144}} e^{-\frac{(t-90)^2}{288}} dt \quad \Phi(z) =$

12

$\boxed{Z = \frac{x-90}{144} \sim N(0,1)}$ * normalização

$\frac{5}{12} \approx 0,41$

a) $P(85 \leq X \leq 95) = P\left(\frac{85-90}{12} \leq \frac{X-90}{12} \leq \frac{95-90}{12}\right) = P\left(-\frac{5}{12} < Z \leq \frac{5}{12}\right)$

$= P\left(Z \leq \frac{5}{12}\right) - \left(1 - P\left(Z < -\frac{5}{12}\right)\right) = 0,66 - (1 - (0,66)) = 0,66 - 0,34$

$= 0,32$ //

o valor da tabela

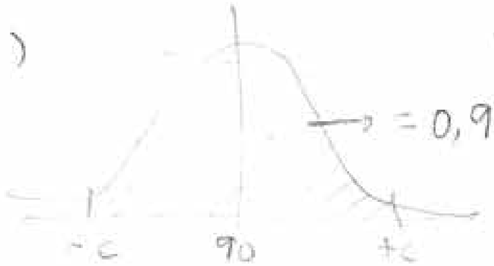
b) $P(78 \leq X < 102) = P\left(\frac{78-90}{12} \leq \frac{X-90}{12} < \frac{102-90}{12}\right)$

$= P(-1 \leq Z \leq 1) = P(Z \leq 1) - P(Z \geq -1)$

$= P(Z \leq 1) - (1 - P(Z < -1)) = 0.8413 - (1 - 0,413)$

$= 0.8413 - 0.1587 = 0,6826$ //

c)



$= 0,95 \Rightarrow P\left(\frac{90-c-90}{12} \leq Z < \frac{90+c-90}{12}\right)$

$= P\left(-\frac{c}{12} \leq Z \leq \frac{c}{12}\right) = 0,95$

$\rightarrow P\left(Z \leq \frac{c}{12}\right) - \left(1 - P\left(Z \leq \frac{c}{12}\right)\right) = 0,95$

$\rightarrow P\left(Z \leq \frac{23}{12}\right) - \left(1 - P\left(Z < \frac{23}{12}\right)\right) \overset{\text{TABELA}}{=} 0,9713 - (1 - 0,9713)$

$= 0,9426$

d) 

$= 1 - P(X < 120)$ $\quad z = \frac{120 - 90}{12} = \frac{30}{12} = 2.5$

lop. 1 -

(D5) Leão $X_1 \sim N(4, \frac{1}{2})$ Tigre $X_2 \sim N(4, \frac{1}{2})$

$$Z = \frac{x-4}{0,5} \sim N(0,1):$$

a) $P(x_1<3 \wedge x_2<3) = P(x_1<3) \times P(x_2<3) = (P(x_1<3))^2$

acontecimentos independentes

$P(x_1<3)$

$\hookrightarrow P(Z<-2) = P(Z \geq 2) = 1 - P(Z<2) = 1 - 0,9772$

$= 0,023^2 = 0.00052$

b) $P(X_1 > X_2)$

d) $P(X_1 > c) = 0,975$

$1 - P(X_1 < c) = 0,975 \Rightarrow P(X_1 < c) = 0.023$

$\hookrightarrow < 0,5$

$$P(X_1 > X_2) = P(\overbrace{X_1 - X_2}^{W} > 0)$$

$$W = \underbrace{X_1 - X_2}_{\text{Propriedades Teorema 2}} \sim N(\mu = 0; \sigma = \sqrt{0,5^2 + 0,5^2} = 0,707)$$

$$Z = \frac{X-0}{0,707}$$

$$P(W>0) = P\left(\frac{W-0}{0,707} > \frac{0-0}{0,707}\right)$$

$$= P(Z>0) = 1 - P(Z \leq 0) = 1 - F_X(0) = 1 - 0,5 = 0,5$$

c) $P\left(\frac{X_1}{2} + \frac{3X_2}{4} > 4\right) =$

Propiedades

$$E[kX] = kE[X]$$

$$V[kX] = k^2 V[X]$$

$$\sigma_{kX} = k\sigma_X$$

$$W = \underbrace{\frac{1}{2}X_1}_{W_1} + \underbrace{\frac{3}{4}X_2}_{W_2}$$

$$W_1 \sim N\left(\mu = \frac{1}{2} \times 4 \; ; \; \sigma = \frac{1}{2} \times 0{,}5\right)$$

$$W_1 \sim N(2 \, , \, 0{,}25)$$

$$W_2 \sim N\left(\mu = \frac{3}{4} \times 4 \; ; \; \sigma = \frac{3}{4} \times 0{,}5\right)$$

$$W_2 \sim N\left(3 \; ; \; \frac{3}{8}\right)$$

$$W \sim N\left(2+3 \; ; \; \frac{1}{4} + \frac{3}{8}\right)$$

$$W \sim N\left(5 \; ; \; \frac{5}{8}\right)$$

$$\boxed{Z = \frac{X-5}{0{,}625}}$$

$$P(Z>4) = 1 - P(Z \leq 4)$$

continuar...