Chapter 11: Indexing and Hashing

Database System Concepts, 6th Ed.

Bitmap Indices

■ Bitmap indices are a special type of index designed for efficient querying on multiple keys

■ Records in a relation are assumed to be numbered sequentially from, say, 0

● Given a number n it must be easy to retrieve record n

▸ Particularly easy if records are of fixed size

▸ Otherwise, a list of pointers might be used

■ Applicable on attributes that take on a relatively small number of discrete distinct values

● E.g. gender, country, state, …

● E.g. income-level (income broken up into a small number of levels such as 0-9999, 10000-19999, 20000-50000, 50000-infinity)

■ A bitmap is simply an array of bits

Bitmap Indices (Cont.)

■ In its simplest form a bitmap index on an attribute has a bitmap for each value of the attribute

- Bitmap has as many bits as records

- In a bitmap for value v, the bit for a record is 1 if the record has the value v for the attribute, and is 0 otherwise

record number	ID	gender	income_level
0	76766	m	L1
1	22222	f	L2
2	12121	f	L1
3	15151	m	L4
4	58583	f	L3

Bitmaps for *gender*

m	10010
f	01101

Bitmaps for *income_level*

L1	10100
L2	01000
L3	00001
L4	00010
L5	00000

Bitmap Indices (Cont.)

- Bitmap indices are useful for queries on multiple attributes
  - not particularly useful for single attribute queries
- Queries are answered using bitmap operations
  - Intersection (and)
  - Union (or)
  - Complementation (not)
  - Example query **where** gender = 'm' and income_level = 'L1'
- Each operation takes two bitmaps of the same size and applies the operation on corresponding bits to get the result bitmap
  - E.g.   100110 AND 110011 = 100010

    100110 OR 110011 = 110111
    NOT 100110 = 011001
  - Males with income level L1:   10010 AND 10100 = 10000
    - Can then retrieve required tuples.
    - Counting number of matching tuples is even faster, not even requiring access to the records in the file

Bitmap Indices (Cont.)

■ Bitmap indices generally very small compared with relation size

- ● E.g. if record is 100 bytes, space for a single bitmap is 1/800 of space used by relation.

  - ▸ If number of distinct attribute values is 8, bitmap is only 1% of relation size

■ Deletion needs to be handled properly

- ● **Existence bitmap** to note if there is a valid record at a record location

- ● Needed for complementation

  - ▸ not($A=v$):   *(NOT bitmap-A-v) AND ExistenceBitmap*

■ Should keep bitmaps for all values, even null value

- ● To correctly handle SQL null semantics for NOT($A=v$):

  - ▸ intersect above result with (NOT *bitmap-A-Null*)

Efficient Implementation of Bitmap Operations

- Bitmaps are packed into words; a single word and (a basic CPU instruction) computes and of 32 or 64 bits at once

  - E.g. 1-million-bit maps can be and-ed with just 31,250 instruction

- Counting number of 1s can be done fast by a trick:

  - Use each byte to index into a precomputed array of 256 elements each storing the count of 1s in the binary representation

    - Can use pairs of bytes to speed up further at a higher memory cost

  - Add up the retrieved counts

- Bitmaps can be used instead of Tuple-ID lists at leaf levels of B^+-trees, for values that have a large number of matching records

  - Worthwhile if > 1/64 of the records have that value, assuming a tuple-id is 64 bits

  - Above technique merges benefits of bitmap and B^+-tree indices

Hashing

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Static Hashing

- A **bucket** is a unit of storage containing one or more records (a bucket is typically a disk block).

- In a **hash file organization** we obtain the bucket of a record directly from its search-key value using a **hash function**.

- Hash function h is a function from the set of all search-key values K to the set of all bucket addresses B.

- Hash function is used to locate records for access, insertion as well as deletion.

- Records with different search-key values may be mapped to the same bucket; thus entire bucket has to be searched sequentially to locate a record.

Example of Hash File Organization

Hash file organization of *instructor* file, using *dept_name* as key
(See figure in next slide.)

- There are 10 buckets,

- The binary representation of the ith character is assumed to be the integer $i.$

- The hash function returns the sum of the binary representations of the characters modulo 10

  - E.g. h(Music) = 1   h(History) = 2
    h(Physics) = 3   h(Elec. Eng.) = 3

Example of Hash File Organization

bucket 0

bucket 1

15151	Mozart	Music	40000

bucket 2

32343	El Said	History	80000
58583	Califieri	History	60000

bucket 3

22222	Einstein	Physics	95000
33456	Gold	Physics	87000
98345	Kim	Elec. Eng.	80000

bucket 4

12121	Wu	Finance	90000
76543	Singh	Finance	80000

bucket 5

76766	Crick	Biology	72000

bucket 6

10101	Srinivasan	Comp. Sci.	65000
45565	Katz	Comp. Sci.	75000
83821	Brandt	Comp. Sci.	92000

bucket 7

Hash file organization of *instructor* file, using *dept_name* as key (see previous slide for details).

Hash Functions

- Worst hash function maps all search-key values to the same bucket; this makes access time proportional to the number of search-key values in the file.

- An ideal hash function is **uniform**, i.e., each bucket is assigned the same number of search-key values from the set of *all* possible values.

- Ideal hash function is **random**, so each bucket will have the same number of records assigned to it irrespective of the *actual distribution* of search-key values in the file.

- Typical hash functions perform computation on the internal binary representation of the search-key.

  - For example, for a string search-key, the binary representations of all the characters in the string could be added and the sum modulo the number of buckets could be returned.

Handling of Bucket Overflows

- Bucket overflow can occur because of
  - Insufficient buckets
  - Skew in distribution of records. This can occur due to two reasons:
    - multiple records have same search-key value
    - chosen hash function produces non-uniform distribution of key values
- Although the probability of bucket overflow can be reduced, it cannot be eliminated; it is handled by using *overflow buckets*.

Handling of Bucket Overflows (Cont.)

■ **Overflow chaining** – the overflow buckets of a given bucket are chained together in a linked list.

■ Above scheme is called **closed hashing**.

● An alternative, called **open hashing**, which does not use overflow buckets, is not suitable for database applications due to the cost of delete operations

overflow buckets for bucket 1
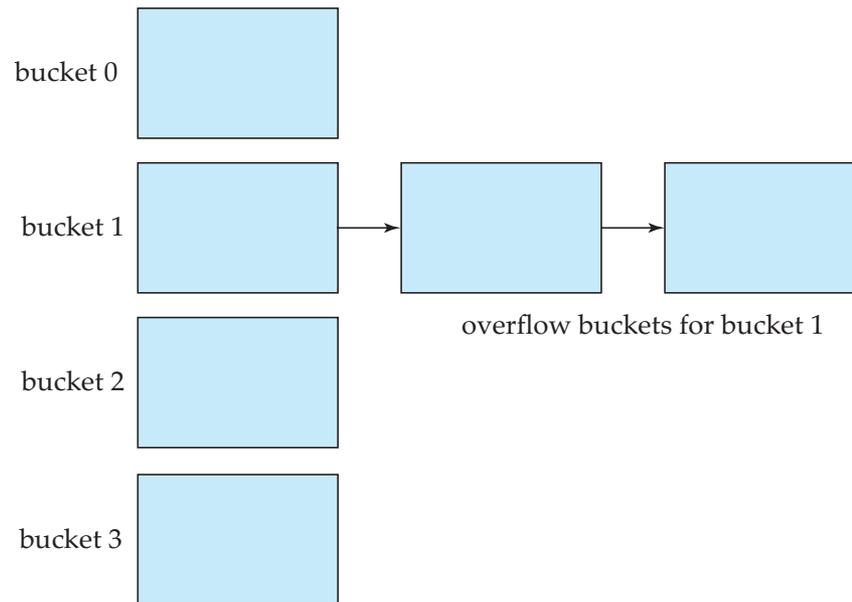
Hash Indices

- Hashing can be used not only for file organization, but also for index-structure creation.

- A **hash index** organizes the search keys, with their associated record pointers, into a hash file structure.

- Strictly speaking, hash indices are always secondary indices

  - if the file itself is organized using hashing, a separate primary hash index on it using the same search-key is unnecessary.

  - However, we use the term hash index to refer to both secondary index structures and hash organized files.

Example of Hash Index

bucket 0			
76766			

bucket 1			
45565			
76543			

bucket 2			
22222			

bucket 3			
10101			

bucket 4			

bucket 5			
15151		58583	
33456		98345	

bucket 6			
83821			

bucket 7			
12121			
32343			

76766	Crick	Biology	72000
10101	Srinivasan	Comp. Sci.	65000
45565	Katz	Comp. Sci.	75000
83821	Brandt	Comp. Sci.	92000
98345	Kim	Elec. Eng.	80000
12121	Wu	Finance	90000
76543	Singh	Finance	80000
32343	El Said	History	60000
58583	Califieri	History	62000
15151	Mozart	Music	40000
22222	Einstein	Physics	95000
33465	Gold	Physics	87000

hash index on *instructor,* on attribute *ID*

Deficiencies of Static Hashing

- In static hashing, function h maps search-key values to a fixed set of B of bucket addresses. Databases grow or shrink with time.
  - If initial number of buckets is too small, and file grows, performance will degrade due to too much overflows.
  - If space is allocated for anticipated growth, a significant amount of space will be wasted initially (and buckets will be underfull).
  - If database shrinks, again space will be wasted.
- One solution: periodic re-organization of the file with a new hash function
  - Expensive, disrupts normal operations
- Better solution: allow the number of buckets to be modified dynamically.

Dynamic Hashing

- Good for database that grows and shrinks in size
- Allows the hash function to be modified dynamically
- **Extendable hashing** – one form of dynamic hashing
  - Hash function generates values over a large range — typically b-bit integers, with $b = 32$.
  - At any time use only a prefix of the hash function to index into a table of bucket addresses.
  - Let the length of the prefix be i bits, $0 \leq i \leq 32$.
    - Bucket address table size = 2^i.   Initially $i = 0$
    - Value of i grows and shrinks as the size of the database grows and shrinks.
  - Multiple entries in the bucket address table may point to a bucket (why?)
  - Thus, actual number of buckets is $< 2^i$
    - The number of buckets also changes dynamically due to coalescing and splitting of buckets.

General Extendable Hash Structure
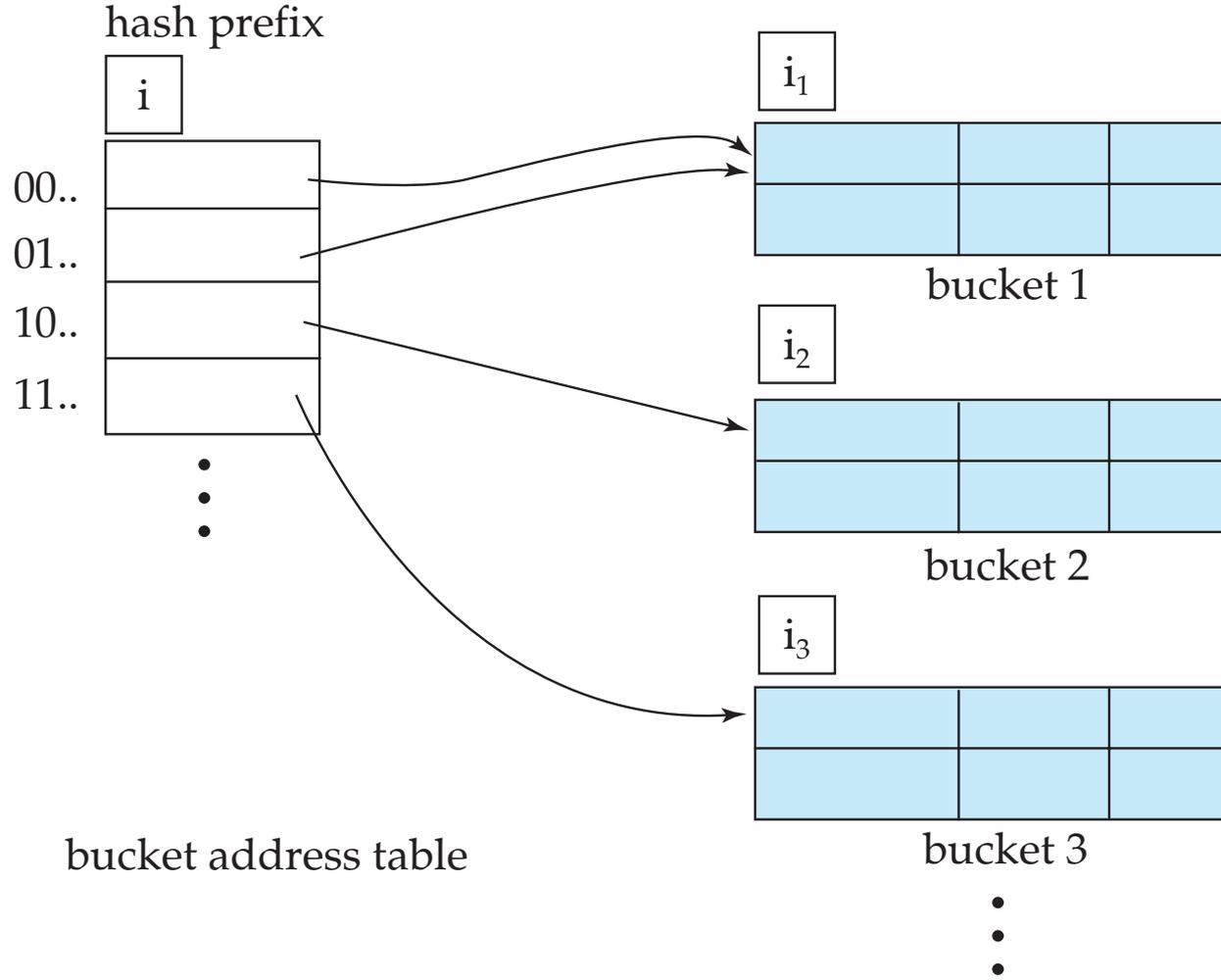
In this structure, $i_2 = i_3 = i$, whereas $i_1 = i - 1$ (see next slide for details)

Use of Extendable Hash Structure

- Each bucket j stores a value i_j

  - *A*ll the entries that point to the same bucket have the same values on the first i_j bits.

- To locate the bucket containing search-key K_j:

  1. Compute $h(K_j) = X$

  2. Use the first i high order bits of X as a displacement into bucket address table, and follow the pointer to appropriate bucket

- To insert a record with search-key value K_j

  - follow same procedure as look-up and locate the bucket, say j.

  - If there is room in the bucket j insert record in the bucket.

  - Else the bucket must be split and insertion re-attempted (next slide.)

    ‣ Overflow buckets used instead in some cases (will see shortly)

Insertion in Extendable Hash Structure (Cont)

To split a bucket j when inserting record with search-key value K_j:

- If $i > i_j$ (more than one pointer to bucket j)
  - allocate a new bucket z, and set $i_j = i_z = (i_j + 1)$
  - Update the second half of the bucket address table entries originally pointing to j, to point to z
  - remove each record in bucket j and reinsert (in j or z)
  - recompute new bucket for K_j and insert record in the bucket (further splitting is required if the bucket is still full)
- If $i = i_j$ (only one pointer to bucket j)
  - If i reaches some limit b, or too many splits have happened in this insertion, create an overflow bucket
  - Else
    - increment i and double the size of the bucket address table.
    - replace each entry in the table by two entries that point to the same bucket.
    - recompute new bucket address table entry for K_j
   Now $i > i_j$ so use the first case above.

Use of Extendable Hash Structure: Example

dept_name	h(*dept_name*)
Biology	0010 1101 1111 1011 0010 1100 0011 0000
Comp. Sci.	1111 0001 0010 0100 1001 0011 0110 1101
Elec. Eng.	0100 0011 1010 1100 1100 0110 1101 1111
Finance	1010 0011 1010 0000 1100 0110 1001 1111
History	1100 0111 1110 1101 1011 1111 0011 1010
Music	0011 0101 1010 0110 1100 1001 1110 1011
Physics	1001 1000 0011 1111 1001 1100 0000 0001

- Initial Hash structure; bucket size = 2

hash prefix

| 0 |

| |
| |

bucket address table

| 0 |
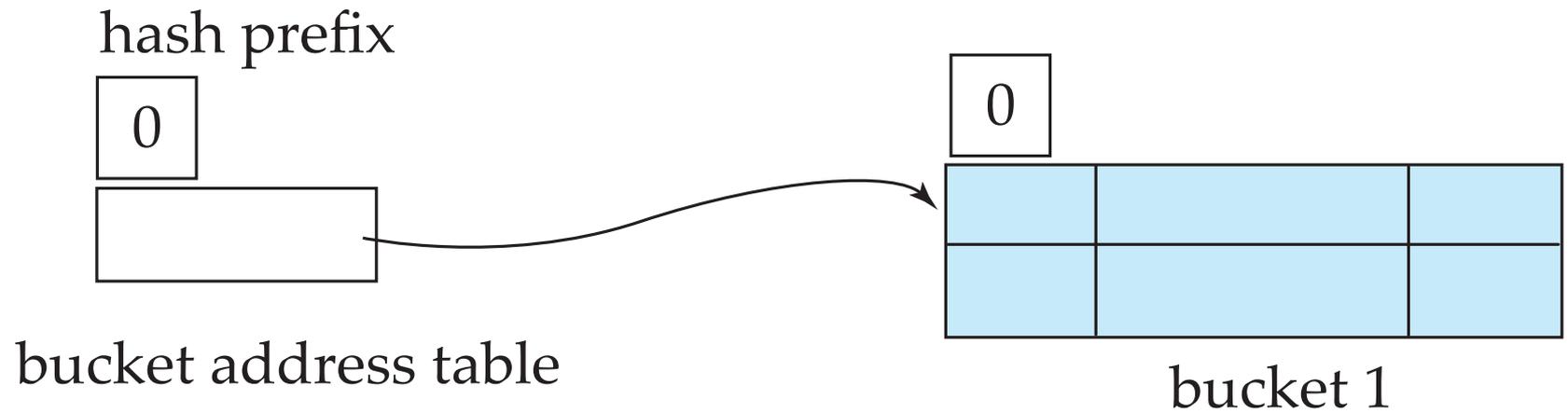
bucket 1

Example (Cont.)

- Hash structure after insertion of "Mozart", "Srinivasan", and "Wu" records

hash prefix

1

0	
1	

bucket address table

1			
15151	Mozart	Music	40000

1			
10101	Srinivasan	Comp. Sci.	90000
12121	Wu	Finance	90000
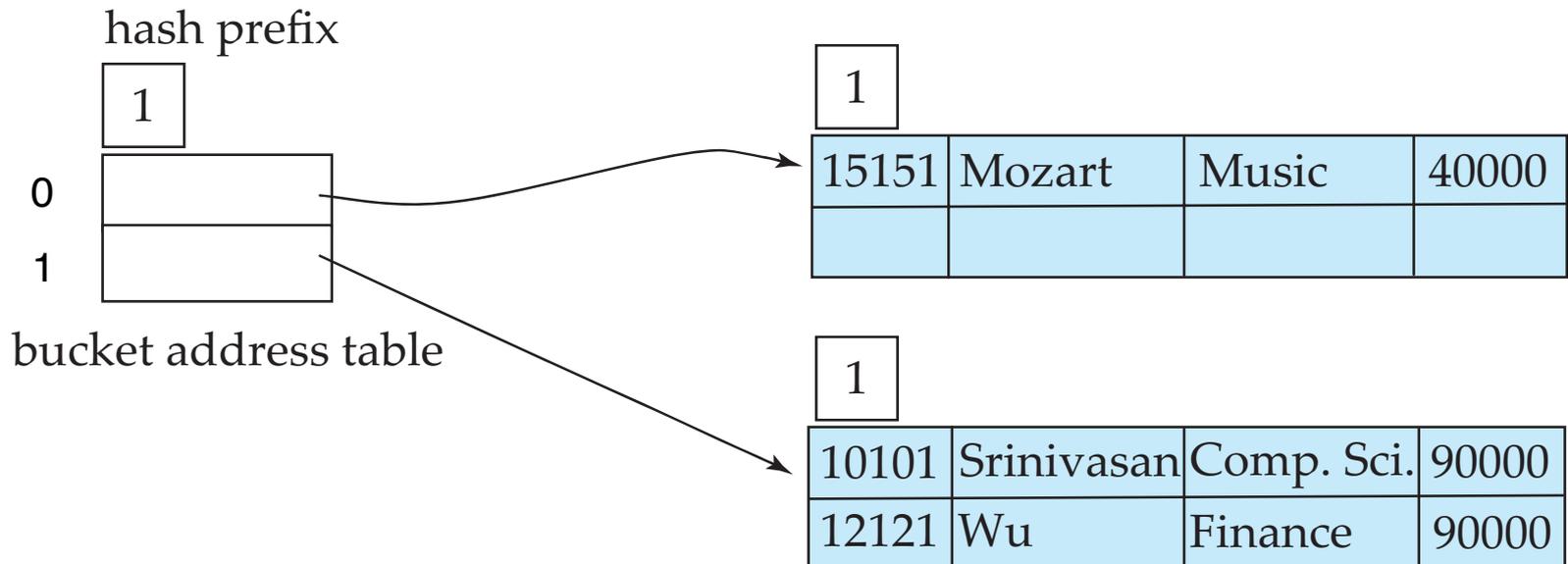
Example (Cont.)

- Inserting Einstein Record (first we double the table…)

hash prefix

2

00	
01	
10	
11	

bucket address table

1

15151	Mozart	Music	40000

1

10101	Srinivasan	Comp. Sci.	90000
12121	Wu	Finance	90000
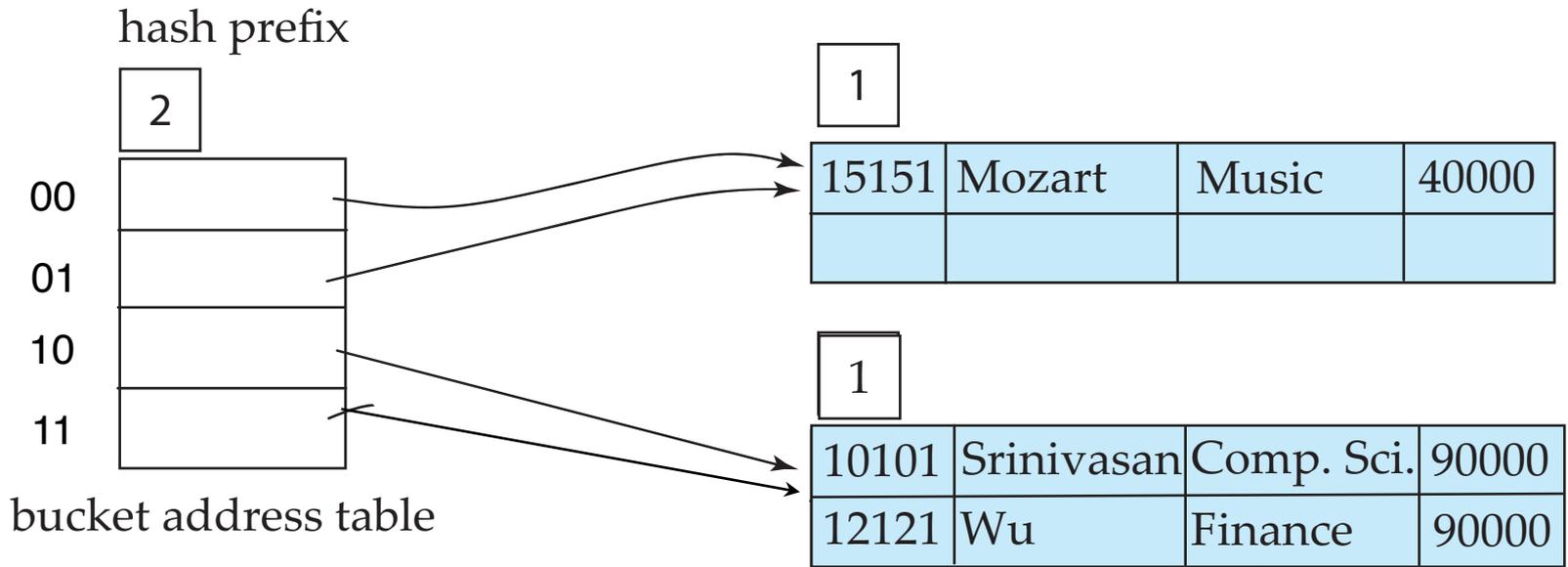
Example (Cont.)

- Then (1st step), a new bucket is allocated (z), $i_z := i_j := i_j + 1$

hash prefix

2

00	
01	
10	
11	

bucket address table

1

15151	Mozart	Music	40000

2 (Bucket j in algorithm)

10101	Srinivasan	Comp. Sci.	90000
12121	Wu	Finance	90000

2 (Bucket z in algorithm)
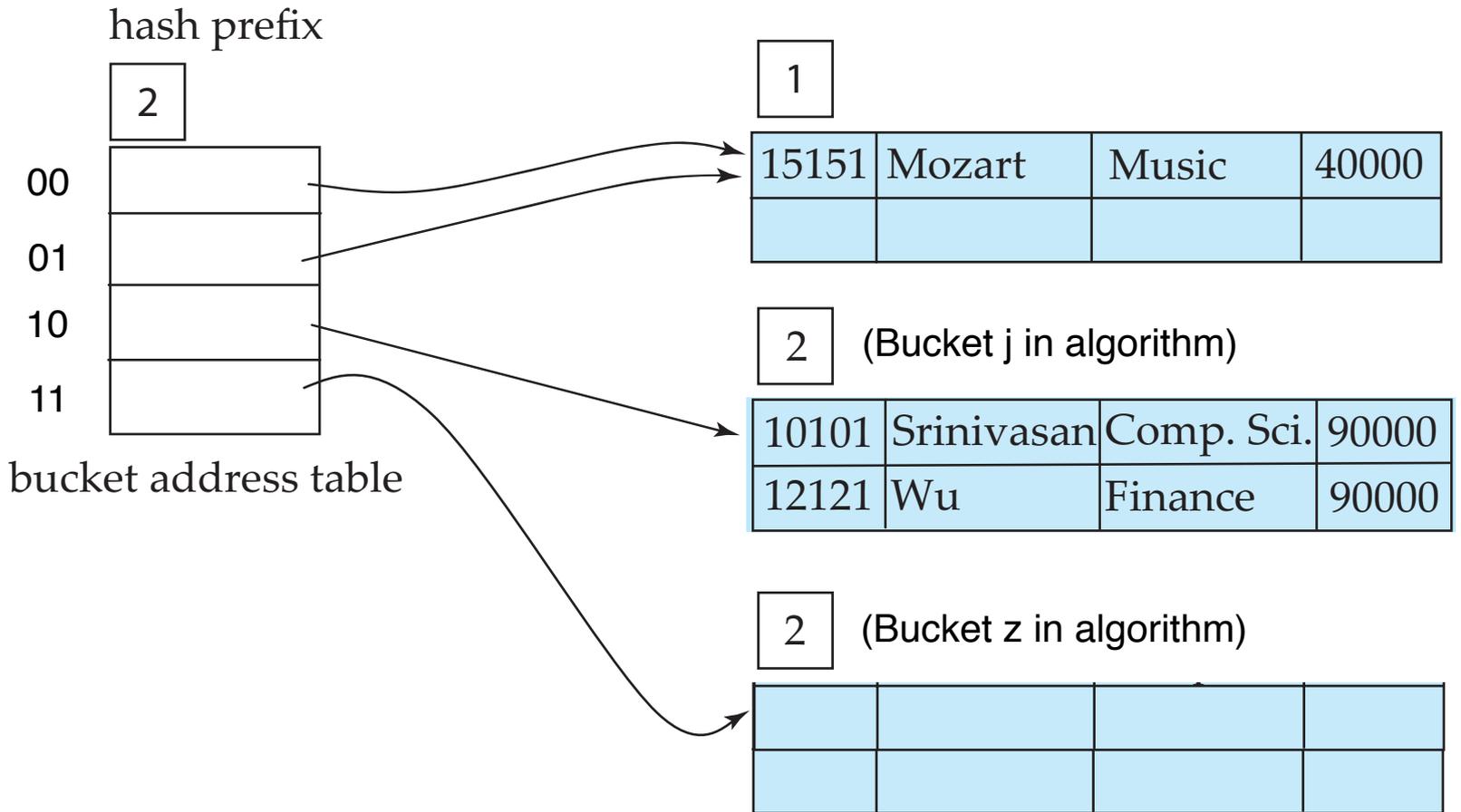
Example (Cont.)

- Reintroduce entries in bucket j (rehash), and insert Einstein record:

hash prefix

| 2 |

	1

bucket address table: 00, 01, 10, 11

| 15151 | Mozart | Music | 40000 |
| | | | |

| 2 | (Bucket j in algorithm) |

| 12121 | Wu | Finance | 90000 |
| 22222 | Einstein | Physics | 95000 |

| 2 |

| 10101 | Srinivasan | Comp. Sci. | 65000 |
| | | | |

dept_name	h(dept_name)
Biology	0010 1101 1111 1011 0010 1100 0011 0000
Comp. Sci.	1111 0001 0010 0100 1001 0011 0110 1101
Elec. Eng.	0100 0011 1010 1100 1100 0110 1101 1111
Finance	1010 0011 1010 0000 1100 0110 1001 1111
History	1100 0111 1110 1101 1011 1111 0011 1010
Music	0011 0101 1010 0110 1100 1001 1110 1011
Physics	1001 1000 0011 1111 1001 1100 0000 0001

Example (Cont.)

■ Hash structure after insertion of Gold and El Said records

hash prefix

3

000	
001	
010	
011	
100	
101	
110	
111	

bucket address table

1

15151	Mozart	Music	40000

3

22222	Einstein	Physics	95000
33456	Gold	Physics	87000

3

12121	Wu	Finance	90000

2

10101	Srinivasan	Comp. Sci.	65000
32343	El Said	History	60000
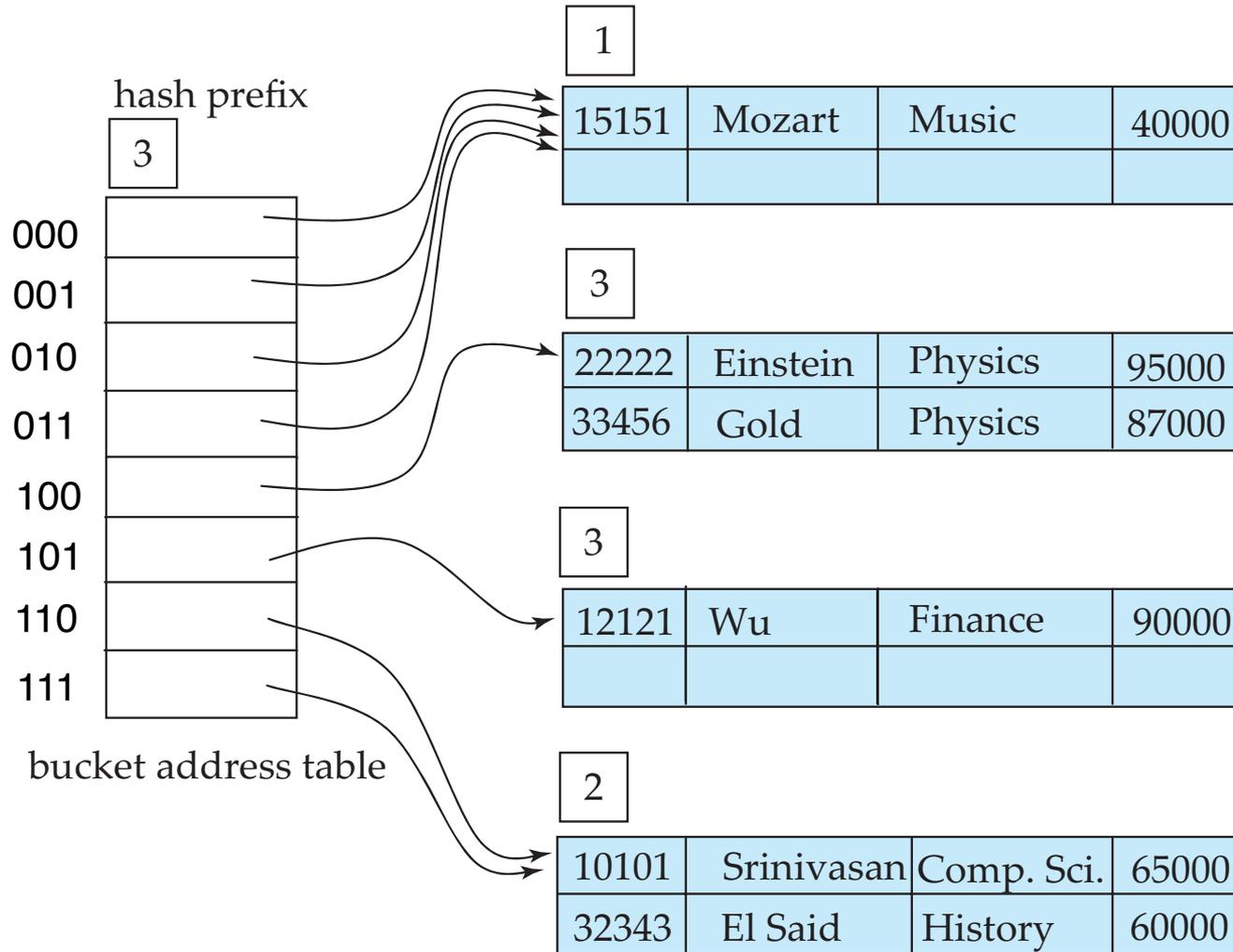
Example (Cont.)

- Hash structure after insertion of Katz record
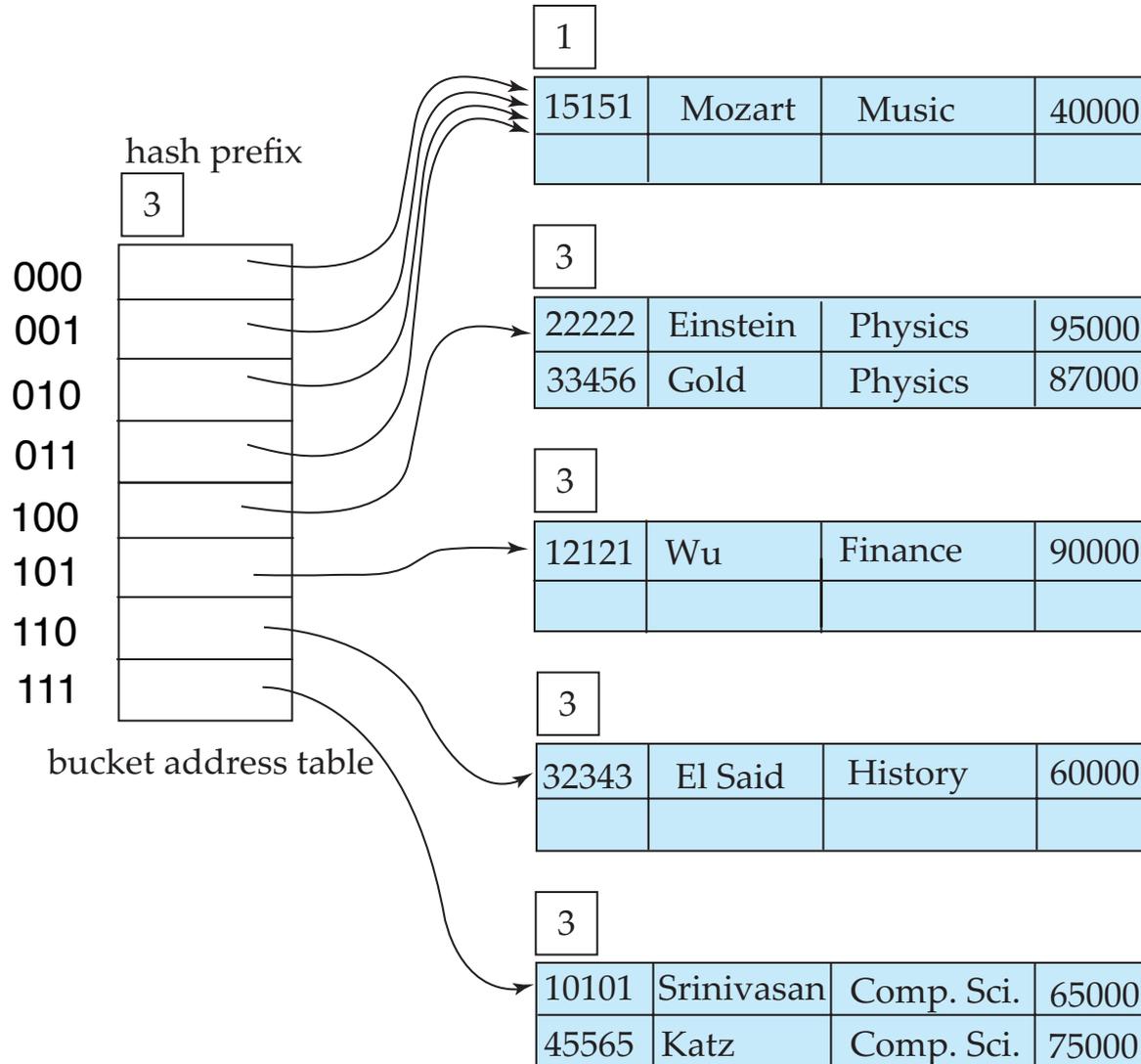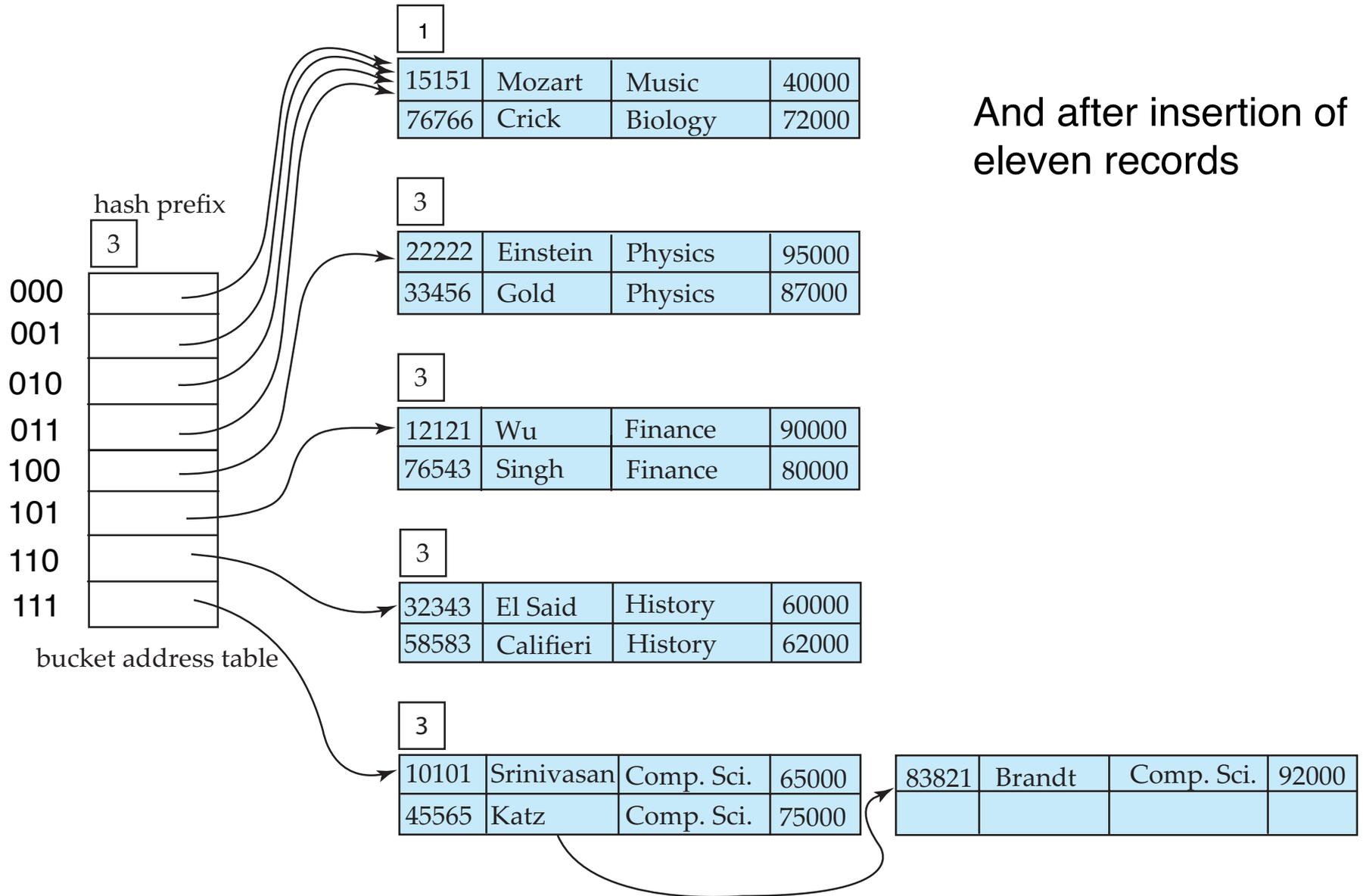
Example (Cont.)

And after insertion of eleven records

Example (Cont.)

2			
15151	Mozart	Music	40000
76766	Crick	Biology	72000

2			
98345	Kim	Elec. Eng.	80000

hash prefix

3

3			
22222	Einstein	Physics	95000
33456	Gold	Physics	87000

3			
12121	Wu	Finance	90000
76543	Singh	Finance	80000

bucket address table

3			
32343	El Said	History	60000
58583	Califieri	History	62000

3			
10101	Srinivasan	Comp. Sci.	65000
45565	Katz	Comp. Sci.	75000

83821	Brandt	Comp. Sci.	92000

And after insertion of
Kim record in previous
hash structure

Deletion in Extendable Hash Structure

- To delete a key value,
    - locate it in its bucket and remove it.
    - The bucket itself can be removed if it becomes empty (with appropriate updates to the bucket address table).
    - Coalescing of buckets can be done (can coalesce only with a "*buddy*" bucket having same value of i_j and same $i_j - 1$ prefix, if it is present)
    - Decreasing bucket address table size is also possible
        - Note: decreasing bucket address table size is an expensive operation and should be done only if number of buckets becomes much smaller than the size of the table

Extendable Hashing vs. Other Schemes

- Benefits of extendable hashing:
  - Hash performance does not degrade with growth of file
  - Minimal space overhead
- Disadvantages of extendable hashing
  - Extra level of indirection to find desired record
  - Bucket address table may itself become very big (larger than memory)
    - ‣ Cannot allocate very large contiguous areas on disk either
    - ‣ Solution: B$^+$-tree structure to locate desired record in bucket address table
  - Changing size of bucket address table is an expensive operation
- Linear hashing is an alternative mechanism
  - Allows incremental growth of its directory (equivalent to bucket address table)
  - At the cost of more bucket overflows

Comparison of Ordered Indexing and Hashing

- Cost of periodic re-organization

- Relative frequency of insertions and deletions

- Is it desirable to optimize average access time at the expense of worst-case access time?

- Expected type of queries:

  - Hashing is generally better at retrieving records having a specified value of the key.

  - If range queries are common, ordered indices are to be preferred

- In practice:

  - PostgreSQL supports hash indices, but discourages use due to poor performance

  - Oracle supports static hash organization, but not hash indices

  - SQLServer supports only B$^+$-trees

Index Definition in SQL

- Create an index

  create index <index-name> **on** <relation-name>
  (<attribute-list>)

  E.g.: **create index** *b-index* **on** *branch(branch_name)*

- Use **create unique index** to indirectly specify and enforce the condition that the search key is a candidate key is a candidate key.

  - Not really required if SQL **unique** integrity constraint is supported

- To drop an index

  drop index <index-name>

- Most database systems allow specification of type of index, and clustering.

Index Definition in Oracle

- Oracle supports B+-Tree indices as a default for the **create index** SQL command

    - B+-Tree indices are created by default for every primary key and unique declaration

- A new non-null attribute *row-id* is added to all indices, so as to guarantee that all search keys are unique.

    - Indices are supported on

        - attributes, and attribute lists;

        - on results of functions over attributes

        - or using structures external to Oracle (domain indices)

- Bitmap indices are also supported, but for that an explicit declaration is needed:

    create bitmap index <index-name>
    on <relation-name> (<attribute-list>)

Hashing in Oracle

- Hash indices are not supported

- However (limited) static hash file organisation is supported for partitions
  create table ... partition by hash(<attribute-list>)
          partitions <N>
          stored in (<tables>)

- Index files can also be partitioned using hash function

  create index ... global partition by hash(<attribute-list>)
  partitions <N>

  - This creates a global indexed partitioned by the hash function

- (Global) indexing over hash partitioned table is also possible

- Hashing may also be used to organise clusters in multitable clusters

End of Chapter

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Figure 11.01

|				
10101	Srinivasan	Comp. Sci.	65000	
12121	Wu	Finance	90000	
15151	Mozart	Music	40000	
22222	Einstein	Physics	95000	
32343	El Said	History	60000	
33456	Gold	Physics	87000	
45565	Katz	Comp. Sci.	75000	
58583	Califieri	History	62000	
76543	Singh	Finance	80000	
76766	Crick	Biology	72000	
83821	Brandt	Comp. Sci.	92000	
98345	Kim	Elec. Eng.	80000	

Figure 11.15
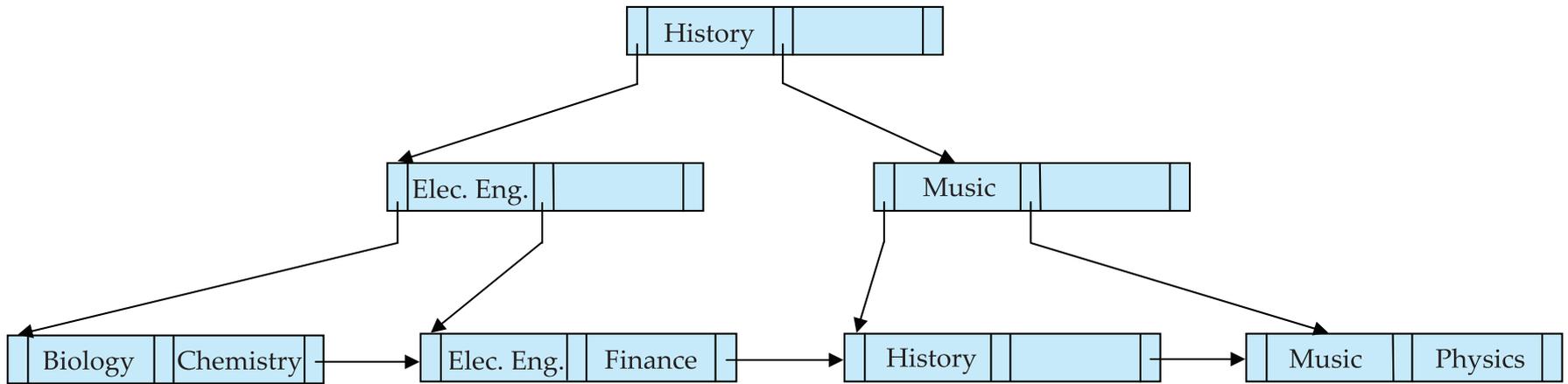
Partitioned Hashing

■ Hash values are split into segments that depend on each attribute of the search-key.

$(A_1, A_2, \ldots, A_n)$ for n attribute search-key

■ Example: $n = 2$, for *customer*, search-key being (*customer-street, customer-city*)

search-key value	hash value
(Main, Harrison)	101 111
(Main, Brooklyn)	101 001
(Park, Palo Alto)	010 010
(Spring, Brooklyn)	001 001
(Alma, Palo Alto)	110 010

■ To answer equality query on single attribute, need to look up multiple buckets. Similar in effect to grid files.

Grid Files

- Structure used to speed the processing of general multiple search-key queries involving one or more comparison operators.

- The grid file has a single grid array and one linear scale for each search-key attribute. The grid array has number of dimensions equal to number of search-key attributes.

- Multiple cells of grid array can point to same bucket

- To find the bucket for a search-key value, locate the row and column of its cell using the linear scales and follow pointer

Example Grid File for *account*
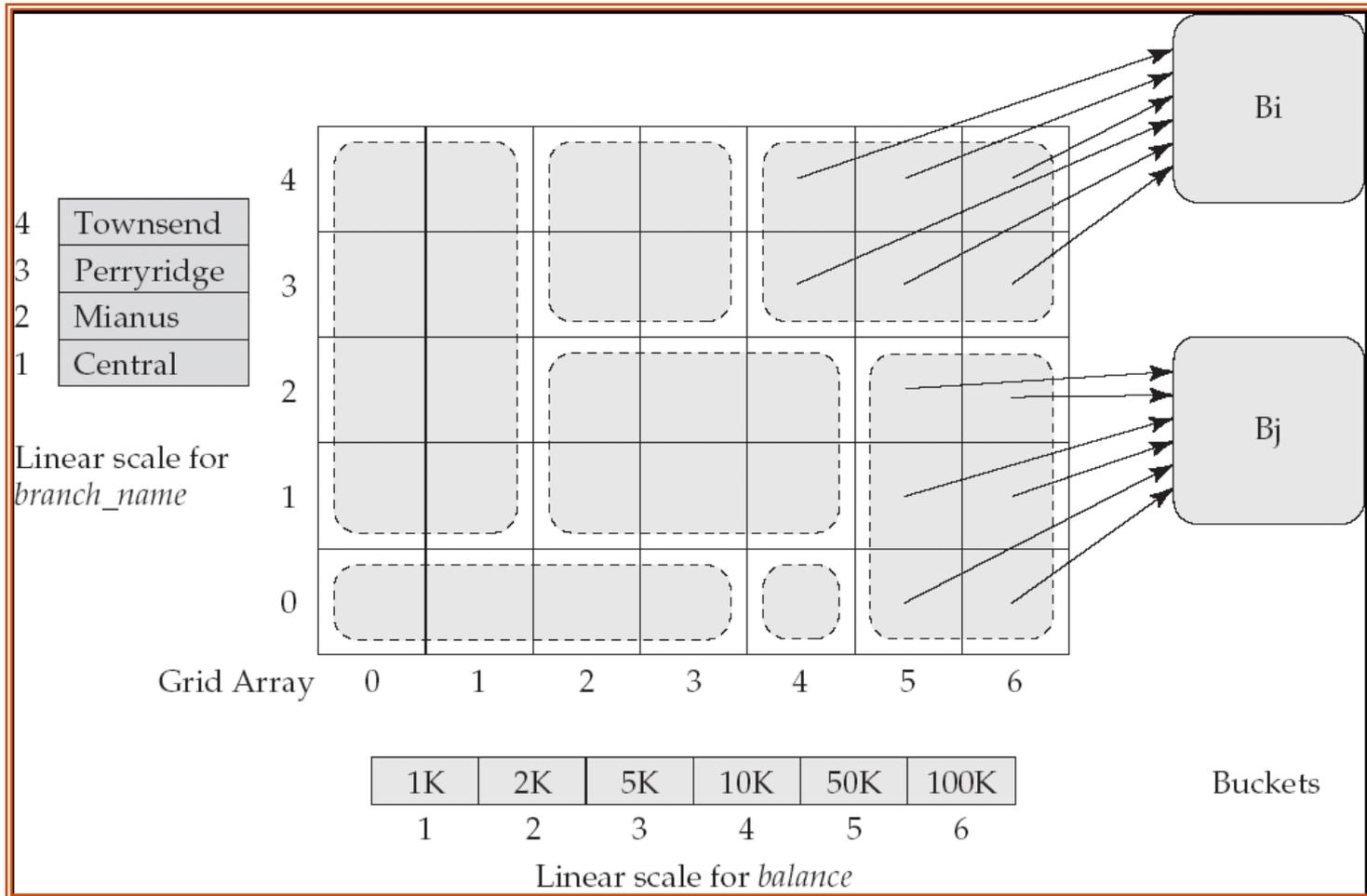
Queries on a Grid File

- A grid file on two attributes A and B can handle queries of all following forms with reasonable efficiency

  - $(a_1 \leq A \leq a_2)$

  - $(b_1 \leq B \leq b_2)$

  - $(a_1 \leq A \leq a_2 \ \wedge \ b_1 \leq B \leq b_2)$,.

- E.g., to answer $(a_1 \leq A \leq a_2 \ \wedge \ b_1 \leq B \leq b_2)$, use linear scales to find corresponding candidate grid array cells, and look up all the buckets pointed to from those cells.

Grid Files (Cont.)

- During insertion, if a bucket becomes full, new bucket can be created if more than one cell points to it.

  - Idea similar to extendable hashing, but on multiple dimensions

  - If only one cell points to it, either an overflow bucket must be created or the grid size must be increased

- Linear scales must be chosen to uniformly distribute records across cells.

  - Otherwise there will be too many overflow buckets.

- Periodic re-organization to increase grid size will help.

  - But reorganization can be very expensive.

- Space overhead of grid array can be high.

- R-trees (Chapter 23) are an alternative