

①

$$a) \{ n \in \mathbb{N} \mid n \% 2 = 0 \}$$

$$b) \{ n \in \mathbb{N} \mid \forall m \in \mathbb{N} \ n \% m = 0 \Rightarrow (m = n \vee m = 1) \}$$

$$c) \{ ((x, y), z) \in (\mathbb{N} \times \mathbb{N}) \times \mathbb{N} \mid z = x + y \}$$

$$d) \text{NOT} \triangleq \{ (TRUE, FALSE), (FALSE, TRUE) \}$$

$$\uparrow (TRUE) = FALSE$$

$$\uparrow (FALSE) = TRUE$$

$$e) \text{AND} \triangleq \{ ((TRUE, TRUE), TRUE), ((TRUE, FALSE), FALSE), ((FALSE, TRUE), FALSE), ((FALSE, FALSE), FALSE) \}$$

$$f) \overline{\text{OP}} \triangleq \{ ((a, b), c) \in (\text{BOOL} \times \text{BOOL}) \times \text{BOOL} \mid ((a', b'), c) \in \text{AND} \wedge \exists a', b', c' \in \text{BOOL} (a, a') \in \text{NOT} \wedge (b, b') \in \text{NOT} \wedge (c, c') \in \text{NOT} \}$$

II

$$a \Rightarrow b = \neg a \vee b = \neg (\neg a \wedge \neg b) = \neg (a \wedge \neg b)$$

$$\text{Implies} \triangleq \{ ((a, b), c) \in (\text{BOOL} \times \text{BOOL}) \times \text{BOOL} \mid ((a, b), c') \in \text{AND} \wedge (b, b') \in \text{NOT} \wedge (c, c') \in \text{NOT} \}$$

$$g) \text{NAME} \triangleq \text{STRING}$$

$$\text{PHONE} \triangleq \{ x \in \text{STRING} \mid x \geq "00000000" \wedge x \leq "99999999" \}$$

$$\text{EMAIL} \triangleq \{ x @ y \mid x, y \in \text{STRING} \}$$

$$\text{CONTACTCARD} = \text{NAME} \times \mathcal{P}(\text{PHONE}) \times \mathcal{P}(\text{EMAIL})$$

$$h) \text{FIND}: \mathcal{P}(\text{CONTACTCARD}) \times \text{NAME} \rightarrow \mathcal{P}(\text{PHONE}) \times \mathcal{P}(\text{EMAIL} \cup \{F\})$$

$$\text{FIND}(c, n) = \begin{cases} \text{FALSE}, & \nexists_{\substack{p \in \mathcal{P}(\text{PHONE}) \\ e \in \mathcal{P}(\text{EMAIL})}} (n, p, e) \in c \\ \text{FINDP}(c, n), & \text{FINDE}(c, n) \end{cases}$$

$$\text{FINDP}(c, n) = \{ p \in \mathcal{P}(\text{PHONE}) \mid \exists_{\substack{(n, p, e) \in c \\ e \in \mathcal{P}(\text{EMAIL})}} \}$$

$$\text{FINDE}(c, n) = \{ e \in \mathcal{P}(\text{EMAIL}) \mid \exists_{p \in \mathcal{P}(\text{PHONE})} (n, p, e) \in c \}$$